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PHYSICAL REVIEW

THE WAVE THEORY OF THE COMPTON EFFECT

By Carl Eckart[1]

Abstract

Change of frequency of a monochromatic light-wave in the neighborhood of moving gravitating matter.—The equation of a light wave in curved four-space is deduced from considerations not directly involving the electromagnetic nature of light. *A wave theory of the Compton effect.* It is shown that whenever a light wave passes through a non-static gravitational field, its frequency must suffer a change. The equations governing this change are formally very similar to Compton's. It is suggested that the peculiarities observed in the interaction of radiation and matter may be due to the existence of non-static gravitational fields in the atom. These results indicate that the possibilities of the wave hypothesis are not yet exhausted.

Introduction

IT HAS been said that it is impossible to arrive at an interpretation of the Compton wave-length change from any wave theory. In the following it will be shown that the wave theory does in general demand a change in frequency whenever radiation passes in the neighborhood of moving gravitating matter. The equations obtained from the wave theory are formally very similar to A. H. Compton's equations for the interaction of an electron and a corpuscle.

The electromagnetic character of the wave does not enter directly into the present treatment. The calculation of the frequency change is therefore merely qualitative. Neither is the motion of the recoil electron predicted explicitly. By introducing the hypothesis that the wave is electromagnetic, the theory could be completed in these respects.

However, if the electromagnetic field equations at present in vogue were made the basis of such a calculation, it seems very improbable that the results would be in agreement with experiment. The author would like to emphasize this fact: that it is the present electromagnetic theory which is untenable, and not the wave hypothesis.

[1] Research Fellow of the Edison Lamp Works (General Electric Company) at Princeton University.

This does not cast any shadow on the brilliant successes of the corpuscular theory of x-ray scattering. That the corpuscular hypothesis and the wave hypothesis are not completely contradictory is evidenced by the history of the theories of light. To take only two specific instances, its finite velocity and aberration have been explained on both hypotheses. The development of the corpuscular theory has almost always preceded the development of the wave theory, and has always proven to be of great heuristic value.

The Equation of a Monochromatic Light Wave In Generally Curved Space

The equation of a monochromatic light wave is usually written

$$S=A \sin 2\pi\nu\left(\frac{x\cos\alpha+y\cos\beta+z\cos\gamma}{c}-t\right).$$

If this equation is postulated to be invariant to the Lorentz transformation, it yields the Doppler principle and the equations for the aberration of light. In the present section, the equation of a monochromatic light wave will be obtained in such a form that it is invariant to any transformation. The first step is easy and obvious; the equation must be of the form

$$S=A \sin 2\pi\varphi \tag{1}$$

where S and A are tensors of the same kind, and φ is itself an invariant. The interpretation of S and A is not considered in the present treatment.

Consider the surface defined by the simultaneous equations

$$\varphi=\text{const}; \quad t=\text{const}.$$

It is the instantaneous locus of all points at which the disturbance has the same phase; in other words, the spatial wave-front. However, the fundamental relativity equation for the propagation of light, $ds=0$, defines the light-path, and not the wave-front. An obvious assumption to make is that the light-paths are everywhere normal to the hypersurfaces $\varphi=\text{const}$. This is the equivalent of certain assumptions made by DeBroglie in a recent paper.[2]

Suppose all the light-paths radiating from the source of disturbance to have been determined. Let the equation of the particular path which passes through the point in four-space on which our attention is focussed, be given in parametric form by[3]

$$x^i=x^i(a)\ .$$

[2] De Broglie, Phil. Mag., (February, 1924)

[3] The use of superscripts to distinguish the coordinates is in better accord with the general conventions of tensor analysis, and it is hoped will replace the more common

Then the assumption made in the last paragraph may be written

$$\frac{\partial \varphi}{\partial x^i} = k g_{ij} \frac{dx^j}{da} . \tag{2}$$

The quantity k will in general be a function of the parameter. By a suitable choice of the parameter, it may be made a constant, however, and this particular parameter is the one we shall indicate by a.

The light-path satisfies the differential equation

$$\left(\frac{ds}{da}\right)^2 = g_{ij} \frac{dx^i}{da} \frac{dx^j}{da} = 0 . \tag{3}$$

Eq. (3) and elementary operations of the tensor calculus applied to Eq. (2) give

$$g^{ij} \frac{\partial \varphi}{\partial x^i} \frac{\partial \varphi}{\partial x^j} = 0 , \tag{2a}$$

which is a differential equation satisfied by φ throughout space.[4]

It remains to be shown that a disturbance given by (1) and (2a) may really be interpreted as a monochromatic wave propagated with the velocity of light. Let the observer be stationed at the point O, and let the coordinates be a natural system at that point. If the disturbance is to be interpreted as wave-motion at all, the observer must write its phase as

$$\varphi = \nu \left(\delta + \frac{dx^1 \cos\alpha + dx^2 \cos\beta + dx^3 \cos\gamma}{V} + dt \right) \tag{4}$$

where $\nu, \delta, \alpha, \beta, \gamma, V$ are constants in the small region surrounding O, and are given their usual significance by the observer at O. On expanding the function φ in terms of the differentials of the coordinates:

$$\varphi = \varphi_0 + \frac{\partial \varphi}{\partial x^i} dx^i \tag{5}$$

and comparing with (4), we get

$$\frac{\partial \varphi}{\partial x^i} = \left(\frac{\nu \cos\alpha}{V} , \frac{\nu \cos\beta}{V} , \frac{\nu \cos\gamma}{V} , \nu \right) . \tag{6}$$

---

subscript. At first there may be some small trouble in distinguishing between exponents and superscripts, but wherever this is likely to arise, as in "x^3 squared," the expression is written $(x^3)^2$.

[4] It should be noted that Eq. (2a) is not the entire equivalent of (2), but is rather a first integral of an equation which would have been obtained if all the parameters had been eliminated from (2).

It must now be shown that V is the velocity of light. From (6) we get

$$V=\frac{\partial\varphi}{\partial x^4}\Big/\sqrt{\left(\frac{\partial\varphi}{\partial x^1}\right)^2+\left(\frac{\partial\varphi}{\partial x^2}\right)^2+\left(\frac{\partial\varphi}{\partial x^3}\right)^2}\,. \tag{7}$$

From Eq. (2a), remembering that the g's are galilean at O, we find that the expression under the radical is equal to $(1/c^2)(\partial\phi/\partial x^4)^2$, whence $V=c$.

The Compton Effect as a Consequence of the Changing Curvature of Space Near a Moving Electron

Since the light-paths are geodesics,

$$\frac{d}{da}\left(g_{ij}\frac{dx^j}{da}\right)-\frac{1}{2}\,\frac{\partial g_{jk}}{\partial x^i}\,\frac{dx^j}{da}\,\frac{dx^k}{da}=0 \tag{8}$$

Differentiating Eq. (2) and substituting from (8),

$$\frac{d}{da}\left(\frac{\partial\varphi}{\partial x^i}\right)=k\frac{d}{da}\left(g_{ij}\frac{dx^j}{da}\right)=\frac{k}{2}\,\frac{\partial g_{jk}}{\partial x^i}\,\frac{dx^j}{da}\,\frac{dx^k}{da}\,. \tag{9}$$

Integrating along the light-path from the point A to the point B

$$\frac{\partial\varphi}{\partial x^i}\Big|_A^B=kg_{ij}\frac{dx^j}{da}\Big|_A^B=\frac{k}{2}\int_A^B\frac{\partial g_{jk}}{\partial x^i}\,\frac{dx^j}{da}\,\frac{dx^k}{da}da\,. \tag{10}$$

If we now substitute from (6), Eqs. (10) become

$$\begin{aligned}
&\frac{\nu_B\cos\alpha_B}{c_B}-\frac{\nu_A\cos\alpha_A}{c_A}=\frac{k}{2}\int_A^B\frac{\partial g_{ij}}{\partial x^1}\,\frac{dx^i}{da}\,\frac{dx^j}{da}da\,,\\
&\quad\vdots\qquad\qquad\vdots\qquad\qquad\vdots\\
&\nu_B\quad-\quad\nu_A\quad=\frac{k}{2}\int_A^B\frac{\partial g_{ij}}{\partial t}\,\frac{dx^i}{da}\,\frac{dx^j}{da}da\,.
\end{aligned} \tag{11}$$

In obtaining this result, it has been assumed that the points A and B are so far removed from any gravitating matter that it is possible to find a set of coordinates which are "isotropic" at both points. Isotropic coordinates are such that the matrix of the g's is,

$$\begin{matrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&-c^2\end{matrix}$$

The coordinates used in (11) must be such a set.

These equations bear a striking formal resemblance to Compton's equations for the wave-length change[5], though the interpretation differs widely. Compton's equations may be written

$$\begin{aligned} \frac{h\nu_B \cos a_B}{c} - \frac{h\nu_A \cos a_A}{c} &= -\frac{mv_x}{\sqrt{1-v^2/c^2}}, \\ \cdot \quad & \cdot \quad \cdot \\ \cdot \quad & \cdot \quad \cdot \\ \cdot \quad & \cdot \quad \cdot \\ \cdot \quad & \cdot \quad \cdot \\ h\nu_B - h\nu_A &= mc^2 - \frac{mc^2}{\sqrt{1-v^2/c^2}}. \end{aligned} \qquad (12)^6$$

In order that $\nu_B - \nu_A \neq 0$, Eqs. (11) demand that $\partial g_{ij}/\partial t \neq 0$, which can only be the case in a non-static gravitational field. In Compton's Eqs. (12) no change in frequency occurs unless $v \neq 0$. This means that the electron must recoil during the act of scattering. The present view interprets the non-static character of the gravitational field to mean the same thing. As remarked in the introduction, the lack of a specific assumption regarding the interaction between the light wave and the matter in its path, makes any more definite prediction of the motion of the recoil electron impossible.

It is difficult to say what the future of the wave hypothesis may be. Its first care must be, of course, to explain the photo-electric effect. This may be possible only by relegating the wave to a subordinate position, as has been done by DeBroglie[2] and by Bohr, Kramers and Slater.[7] On the other hand, the effect of the changing curvature of space on the radiation emitted by the electron has never been taken into account, so far as the author is aware. It would seem worth an investigation to see whether this might possibly be the origin of the quantum equations, though here again it seems useless to proceed until a new electromagnetic theory has been developed.

PALMER PHYSICAL LABORATORY,
PRINCETON UNIVERSITY,
August 13, 1924.

NOTE BY F. W. BUBB

In conversation with Mr. Eckart there developed a very interesting similarity between the recent theory of DeBroglie and the present theory. Both these views essentially regard the frequency of radiation as the time component of a vector whose

[5] A H. Compton, Phys. Rev. **21**, 483 (1923)

[6] It will be observed that Eq. (11) does not contain Planck's constant h, while Compton's equations do. This suggests that h appears whenever electro-magnetic and gravitational fields interact.

[7] Bohr, Kramers and Slater, Phil. Mag., (May 1924)

properties are very similar to those of the covariant energy-momentum vector of a particle. The puzzling questions which arise through the quantum theory's peculiar use of the frequency, and in fact a critical survey of the classical theory itself do not leave one perfectly satisfied with our ordinary conception of frequency. This term "frequency" beats upon our ears from all directions with such high frequency that it is possible we are more familiar with its sound than with its deeper significance. The present attempts (one finds them here and there) to generalize the meaning of frequency are decidedly in order. It seems likely that a proper generalization of the frequency will either answer or remove some of the questions just now tormenting us so.

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THEORY AND EXPERIMENTS RELATING TO THE STRIATED GLOW DISCHARGE IN MERCURY VAPOR

By K. T. Compton, Louis A. Turner and W. H. McCurdy

Abstract

Theory of the glow discharge in a monatomic gas.—For the case of parallel plane electrodes with a hot cathode as source of electrons, the potential distribution and ion concentration in the Crookes dark space, negative glow, Faraday dark space and positive column are shown to be predictable from considerations of space charge and of ionization and excitation of the gas. While with weak currents there is a negative space charge throughout, sufficiently intense ionization is shown to lead to a cathode drop, followed by a region of reversed electric field in which positive ions and electrons both move toward the anode by diffusion, owing to their large concentration gradient. Still farther from the cathode the field changes to its normal direction and increases up to the positive column. In the positive column the field and concentration are uniform unless atoms excited by electron impacts in certain layers are prevented from diffusing between the layers, when *striations* may be obtained with periodic changes of field and of concentration. The cathode edge of each striation has a positive space charge. The *theory of the arc discharge* is essentially the same, the arc being simply the negative glow of the longer glow discharge.

Glow discharge in mercury vapor.—Various predictions of the above theory were verified by experiments with Hg vapor in vacuum tubes provided with hot cathodes. (1) *Potential distribution and ion concentration* were investigated by Langmuir's modified probe method and found to agree with the theoretical deductions, except that the concentration of positive ions in the positive column comes out too large. This result indicates the *presence of negative mercury ions*. (2) *The distribution of velocities of electrons* is Maxwellian except between striations. (3) *The emission of light* seems associated more with excitation by electron collision than with ionization and recombination. (4) *Conditions for existence of striations*. Striations are not found in pure Hg vapor unless the current is small or some substance like H_2 is introduced to remove excited atoms. (5) The presence of atomic hydrogen which should be produced in the process of removing excited Hg atoms was proved by use of tungsten oxide. (6) Introduction of He, which cannot remove excited atoms, does not tend to produce striations. (7) *The relative concentrations of excited atoms* was determined from the optical absorption of subordinate series Hg lines. It was found that excited atoms exist in striations but not in the regions between, and are more numerous if the amount of H_2 impurity is reduced.

Band spectrum of HgH seems associated with the action of excited Hg atoms on hydrogen, and is emitted as a result of inelastic collisions in striations.

A. Theoretical

THEORETICALLY it should be possible to derive an equation to describe completely the phenomena in glow discharges. Sir J. J. Thomson outlined a method of attack in which he considered the effects

of ionization, recombination, mobility, diffusion and space charge.[1] Unfortunately he was able to obtain solutions of the equations only in very restricted cases, even though the equations were much less general in form than they should be to include a number of phenomena which have only recently been understood.

We have, therefore, made an attempt to apply purely physical considerations of a rather qualitative nature and have succeeded in arriving at certain conclusions which are supported by experimental tests and which give an apparently satisfactory explanation of the essential features of the glow discharge in mercury vapor. Probably similar considerations hold for other monatomic gases.

We shall suppose that our discharge tube is supplied with a hot cathode as a source of electrons, in order to reduce the problem to its simplest terms. This avoids the complication present in dealing with a cold cathode, arising from the necessity of a cathode drop which is sufficiently large to cause emission of electrons from the cathode by positive ion bombardment in sufficient number to maintain the discharge. Our case depends only on the properties of the gas and presents all the essential features of glow discharges.

(1) Cathode Drop, Negative Glow, Faraday Dark Space

Consider first the effect of ionization upon the potential distribution between a hot cathode source of electrons and some region of the gas distant d (such as a near-by anode) maintained at a positive potential V_a which is greater than the ionizing potential of the gas V_i. For the sake of simplicity we shall consider plane parallel electrodes; similar reasoning would apply to electrodes of any shape.

Curve 1, Fig. 1, shows the linear distribution of potential which would exist in the absence of space charge, i. e. if the electron emission from the cathode were suppressed. Curve 2 shows the distribution which would exist if the electron emission occurred as limited by its own space charge but with ionization suppressed. The existence of negative space charge results in a positive curvature at every point, and the exact shape of the curve is determined by the density of this space charge at every point, together with the fact that the initial and final points are fixed and the curve is horizontal at the origin.

Suppose, now, that a definite small amount of ionization is allowed at the point where the electrons have fallen through the ionizing potential V_i. The positive ions thus formed will be drawn toward the cathode

[1] J. J. Thomson, Conduction of Electricity through Gases.

and will neutralize the negative space charge of $A\sqrt{M/m}$ times their number of electrons, where A is a constant greater than, but of the order of, unity. A would be $4\sqrt{2}$ if positive ions and electrons were in thermal equilibrium in the electric field,[2] it would equal $2^{5/4}$ if both collided inelastically at every impact with a gas molecule.[3] In general $A\sqrt{M/m}$ is equal to the ratio of the average velocity of advance of the electrons to that of the positive ions and will have different values in different parts of the discharge, except in the case of a very long discharge with uniform potential gradient. In particular, very close to the cathode the

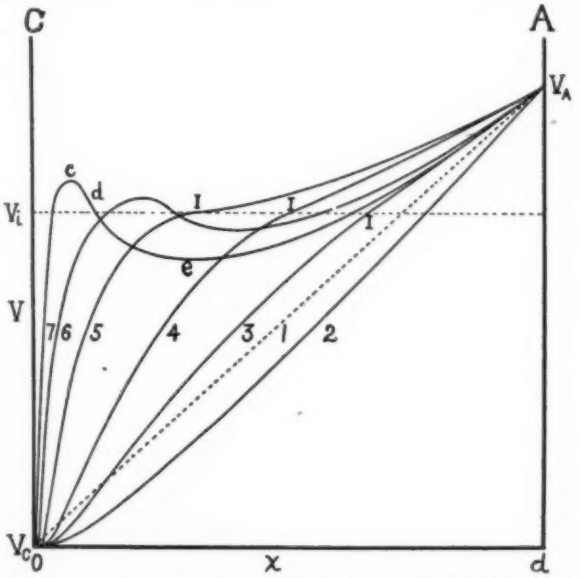

Fig. 1. The effect of increasing amounts of ionization on the distribution of potential between two parallel electrodes, one of which is a source of electrons. Curve 1 shows the electrostatic distribution, curve 2 the distribution in the presence of space charge from the electrons, and the remaining curves the distributions in cases of successively increasing amounts of ionization at the ionizing potential V_i.

electrons are moving away relatively slowly, while the positive ions are moving rapidly, so that the value of $A\sqrt{M/m}$ diminishes to its minimum value as the cathode is approached. Now it is just at the surface of the cathode that the limitation of emission by space charge is effective, the current being such as to make the electric intensity zero at this point.[4]

[2] K. T. Compton, Phys. Rev. **21**, 276 (1923)

[3] K. T. Compton, Phys. Rev. **7**, 492 (1916), equation (9)

[4] We are neglecting corrections due to initial velocity of emission (Langmuir, Phys. Rev. **21**, 419, 1923) which would complicate the argument but would not change the conclusions to be reached.

Thus the increase in the electron emission which results from the ionization is determined by the value of $A\sqrt{M/m}$ near the surface of the cathode, being just sufficient to neutralize the charge of the positive ions there, but insufficient to neutralize it in regions farther away. The final result, therefore, is to leave a zero electric intensity and a negative space charge at the surface of the cathode, with a diminishing negative space charge, which may be replaced by a positive space charge if the ionization is sufficient, at greater distances. Such a condition is shown by Curve 3, in which the space charge is positive (curvature negative) between the point I at which ionization occurs and another point closer to the cathode. Between I and the anode the space charge is negative, and more so than in Curve 2 because of the larger electron current. Since the curvature is governed by the space charge at each point and the two end points are fixed, the potential distribution curve is uniquely determined. The result of the ionization is to concentrate most of the potential drop into a region nearer the cathode.

If the amount of ionization be increased, these changes in the potential distribution will be magnified, as shown in Curves 4 and 5. Although the potential gradient between the point I and the anode is diminished, the concentration of electrons must be more than proportionally increased, so as to carry the increased current.

Further increase in the amount of ionization leads inevitably to a potential distribution of the type of Curves 6 and 7, for increased ionization gives increased space charge and hence increased curvature. With fixed end points, large negative curvature near the cathode and large positive curvature near the anode can be satisfied only by a line with a maximum and a minimum.

At first sight the possibility of a reverse electric field is difficult to accept, and it is quite at variance with the theoretical attempts which have hitherto been made to explain the discharge. In the region of the negative field the current is obviously not due to the electric field; it can only be a *diffusion* current caused by a large concentration gradient of ions, both positive and negative. In earlier theories of the discharges, diffusion has been considered to be of secondary importance and generally neglected in order to simplify the equations. The actual explanation of this feature of the discharge appears to be as follows:

The region of ionization is a region of maximum concentration of ions of both signs. There exists, therefore, a force due to a partial pressure gradient proportional to dn/dx (where n is the number of ions of either sign per unit volume) tending to move the ions toward the regions of less

concentration. The magnitude of this force acting on the ions in unit volume is $-kT(dn/dx)$, where kT is two-thirds the mean kinetic energy of the ions. At the same time there is a force on these ions, due to the electric field E, equal to neE. The resultant force on the ions of either sign, per unit volume, is

$$F = neE - kT(dn/dx). \tag{1}$$

Between the region of ionization and the cathode these two forces are always in the same direction for positive ions and in opposite directions for electrons, but the first term must always predominate, else the discharge would cease.

Between the region of ionization and the anode, the conditions are more complicated. If the ionization is weak, the second term is small, all electrons move toward the anode and no positive ions enter this region. If the amount of ionization is increased, the first term is diminished because of the concentration of potential drop near the cathode, while the term due to diffusion is increased. With sufficiently intense ionization, the force due to diffusion exceeds that due to the electric field and we then have *both electrons and positive ions drifting toward the anode from the region of ionization*. This latter condition tends to produce a reverse electric field in the region of the concentration gradient, on account of the greater mobility of the electrons. We have, in effect, a diffusion cell[5] in which the electrons drift faster than the positive ions towards the anode and set up a negative space charge near the anode, leaving a positive space charge behind, and setting up an electric field which accelerates the positive ions and retards the electrons so that they drift at the same rate. If the positive ions and electrons are present in concentrations which are large compared with their difference (as is shown experimentally to be true in this region, by interpreting the potential gradient by Poisson's equation), this reverse electric field is of such magnitude as to give a potential difference

$$V_1 - V_2 = \frac{kT}{e}\frac{\mu_- - \mu_+}{\mu_- + \mu_+}\log\frac{n_1}{n_2} \tag{2}$$

between any two points 1 and 2. μ_- and μ_+ are the mobilities of the electrons and positive ions, respectively.

Thus far we have not considered recombination between electrons and positive ions. The rate of recombination is proportional to the product of the concentrations of the electrons and positive ions, and is therefore maximum in the region of maximum concentration c, Curve 7.

[5] Wood, de Long and Compton, Phil. Mag. **32**, 499 (1916)

As the ions diffuse toward the anode, they continually recombine and thus the concentration gradient is larger than it would otherwise be. Equation (2) still holds, but the ratio n_1/n_2 is increased by recombination.

Now there must always be an *excess* of current carried by electrons, to account for the current actually flowing in the tube. Near the point *c*, Curve 7, this excess is small compared with the total amount of electricity of both signs moving toward the anode. Farther on, toward *d*, the total ion concentration has been reduced by recombination, and as the anode is further approached the positive ions have so largely disappeared by recombination that the space charge becomes negative at *d*. From this point to the anode the potential distribution curve has positive curvature and the field is in the normal direction beyond a point of minimum potential, *e*.

Such is the physical picture of the phenomena near the cathode which seems to be the inevitable consequence of the properties of the ions. The only condition necessary to secure this type of discharge is intense ionization, which is insured by a sufficient applied potential difference, a cathode sufficiently hot to supply the electrons and a gas at any pressure not so low as to give too infrequent collisions nor so high as to cause electrons to reach their terminal speeds at speeds below that necessary for ionization.[6] These are the conditions for a glow discharge. When they are not met we have a discharge with very much smaller current and negative space charge throughout.

The cathode drop is the potential drop between the cathode and the region of maximum ion concentration. It must exceed the lowest potential at which the gas can be ionized. The negative glow is that region in which recombination is rapid. The glow is therefore of maximum intensity in the region of maximum ion concentration and fades out gradually toward the anode. The Faraday dark space is simply that region in which the glow from recombination is faint, owing to paucity of ions. The Crookes dark space is the region near the cathode in which recombination is infrequent on account of the strong field and the high speed of the ions. A confirmation of the above distinction, or lack of distinction, between the negative glow and the Faraday dark space is found in the fact that no abrupt change in intensity of spectral lines is found as they are followed out from the one region into the other.

Oscillating Discharge

Consideration of conditions immediately outside the cathode leads to the conclusion that the current through the discharge may be of an

[6] Compton, Phys. Rev. **22**, 333 (1923)

oscillatory or interrupted nature if the external resistance in series with the discharge is large and the ionization intense. This is evident from the fact that, if the ionization is so intense as to give a supply of positive ions sufficient to create a positive space charge clear up to the surface of the cathode, then the electron emission necessarily increases to the saturation value determined by the cathode temperature, and the increased potential drop across the series resistance may reduce the drop across the discharge to a value too small to maintain it. The current then diminishes, with redistribution of potential, the discharge builds up again, and the cycle is repeated. Such a process is discussed in some detail for the case of low voltage arcs in a recent paper.[7]

(2) Positive Column and Striations

Now consider the region nearer the anode in a long discharge tube, Fig. 2. We have seen that the big concentration gradient to the right of *c* diminishes as we proceed toward the anode and the positive ions disappear through recombination, leaving, beyond *d*, a negative space charge and a potential distribution curve which is concave upward. At some point along this curve, such as *f*, the electrons must again acquire sufficient energy to ionize the gas. This is inevitable because of the continual increase of electric field shown by the positive curvature. It is also necessary in order to satisfy Poisson's equation, whose application shows that positive and negative ions must exist in nearly equal concentrations at any given point in the discharge, except within the cathode drop. Thus the electric field increases toward the anode until at some point *f* it is sufficient to cause just enough ionization to compensate for the loss of positive ions through recombination and diffusion to the walls of the tube. In this equilibrium condition the concentrations of positive ions and electrons must be equal, so that the potential distribution curve is straight and continues with constant gradient up to the close vicinity of the anode. This region *f-a* is the positive column. At some point between *d* and *f* the concentration of positive ions reaches a minimum.

Under certain conditions, to be discussed later, the positive column is striated. We venture to propose the following explanation of the striations, based upon the effect of inelastic impacts upon the mobility of the electrons and, therefore, upon the potential distribution.

[7] Eckart and Compton, Phys. Rev. **24**, 97, 1924.

Suppose that an electron collides inelastically with a molecule at some point such as h, Fig. 2. Now the mobility of an electron is given by the expression[8]

$$\mu_- = 0.815\,(el/mc)\,, \tag{3}$$

where l is the mean free path and c is the root mean square translational speed. Evidently the electron mobility is minimum just before it has acquired sufficient speed to collide inelastically and maximum immediately after this collision. The contribution of the electron to the negative space charge varies inversely as its mobility, so that the effect of

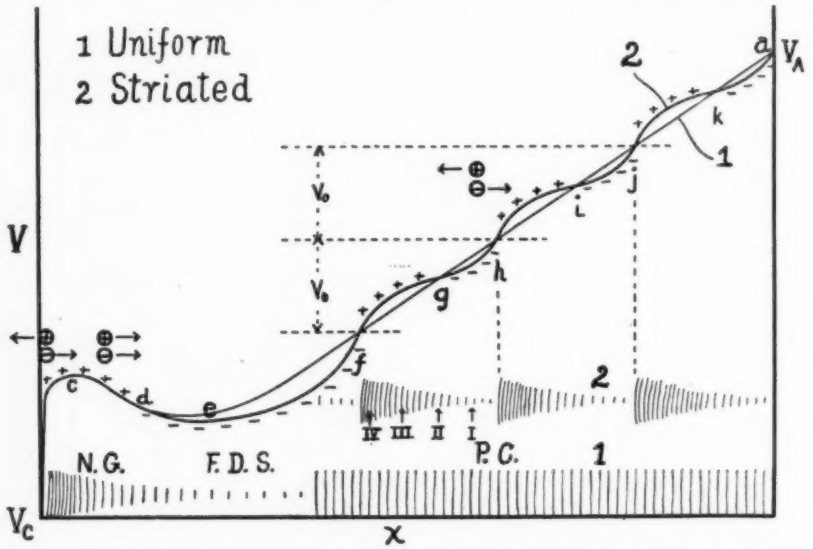

Fig. 2. The theoretical potential distribution in the case of a long discharge tube and the effect of inelastic impacts at points f, h and j on the potential distribution and space charge. Arrows indicate the direction of average drift of ions in various parts of the discharge.

the inelastic collision is to cause an abrupt decrease in negative space charge. As the electron again acquires speed, its mobility diminishes, its contribution to the negative space charge increases, and finally the process is repeated at another inelastic impact nearer the anode. Since the positive column is, on the average, a region of zero space charge, these variations produced by successive inelastic impacts must give rise to alternate regions of positive and negative space charge, as indicated by the heavy curve of Fig. 2.

The important feature of these variations in electric intensity is that they are such as to increase the probability that other electrons will

[8] Townsend, Electricity in Gases, p. 84.

collide inelastically in the same regions. This is because of the increased electric intensity in the region of an inelastic impact. Thus we have the condition that one inelastic impact tends to cause other inelastic impacts in the same region, which is just what is needed to account for striations. Given, therefore, electrons moving in a field through a gas in which impacts are elastic below a certain speed and inelastic above it, we are led to the conclusion that there will be successive regions in which the inelastic impacts will be concentrated, that the potential drop between these regions will be that necessary to enable the electrons to acquire the critical speed for inelastic impact and that the regions of inelastic impacts will be characterized by positive space charge and will be separated by regions of negative space charge. Since inelastic impacts produce excitation or ionization of the gas molecules, the regions of inelastic impact will be regions of maximum luminosity. It should also be noted that the production of positive ions by inelastic impacts adds to the positive space charge of the region in which they occur, thus accentuating the influence of the inelastic impacts. On account of recombination, the positive ions are present in deficient numbers in the intervening negatively charged regions. This effect of the positive ions, however, is *not essential* to the existence of striations, as shown by Grotrian's[9] observation of striations in mercury vapor in the absence of ionization.

Thus, from fundamental properties of electrons, ions and molecules, we have been led to infer a potential distribution as indicated in Fig. 2, and to conclusions regarding distribution of luminosity, space charge and ion concentration. The experiments to be described below confirm these inferences in every detail.

Complicating Effect of Excited Atoms

An additional factor enters to complicate the situation in the case of a gas, like mercury vapor, which may exist in excited states, especially if some of these states are metastable or otherwise of long duration. Suppose that electrons collide inelastically with normal mercury atoms in the region h, Fig. 2, and put these atoms in an excited state. Those electrons cannot again collide inelastically with normal mercury atoms until they have moved to j, distant one critical potential difference from h. But if the excited atoms produced at h and j can diffuse into the intervening space, electrons may collide inelastically with them almost anywhere in this space, since there are a large number of stages of higher excitation into which the previously excited atoms may be put at another impact,

[9] Grotrian, Zeits. f. Phys. **5**, 148 (1921)

and some of these may be produced after the electron has fallen through only a small potential drop. Thus, whereas inelastic impacts can occur only at intervals of a critical potential in the case of normal atoms alone, they may occur almost anywhere if numerous excited atoms are present in addition to the normal atoms. In order to have sharply defined striations, therefore, it is necessary to have so small a number of excited atoms between the striations as to make it improbable that electrons will collide with them. Such a small concentration of excited atoms is realized either by using very small currents or by mixing with the gas some other gas which has the property of causing the disappearance of excited atoms. Both of these deductions are confirmed by the following experiments with mercury vapor.

B. Experimental

In order to test the foregoing theory the following series of experiments were performed. The tests of the last part of the theory will be described first.

(1) Effect of Excited Atoms on Existence of Striations

It was immediately found, as had been known before in other cases, that the positive column in mercury vapor is *uniform* if the vapor is free from impurities (except in the particular case of currents too small to give a cathode drop, in which case the luminosity in the striations is the continuous band spectrum of Hg_2 arising from temporary molecular combinations of excited mercury atoms[10]).

If a trace of hydrogen is added to the mercury vapor, striations appear if the current is low but spread out and blend into a uniform positive column as the current is increased. If more hydrogen be added, the striations persist to larger currents, but ultimately merge as before when the current is further increased.

In order to show that the action of the hydrogen is really to reduce the concentration of excited atoms, an experiment was carried out in which the concentration of excited atoms in various parts of the discharge was tested by their optical absorption of sharp and diffuse series lines of a mercury arc. The arrangement is shown in Fig. 3.

Light from a cooled quartz mercury arc Q was focussed by a quartz lens L_1 on the axis of the quartz tube AF in which the discharge was maintained in mercury vapor at any convenient pressure, which could be

[10] Grotrian, loc. cit.[9];
Franck and Grotrian, Zeits. f. Phys. **4**, 89 (1921)

regulated by the temperature of a surrounding electric oven, not shown. The light passing through the tube was focussed on the slit of a Hilger quartz spectrograph, and its spectrum photographed. Comparison of alternate exposures with the discharge in AF first on and then off, indicated that the subordinate series lines ending on $2p_1$, $2p_2$ and $2p_3$ were strongly, and about equally, absorbed by the luminous vapor, but not by the unexcited vapor. The degree of this absorption is a measure of the concentration of excited atoms.[11]

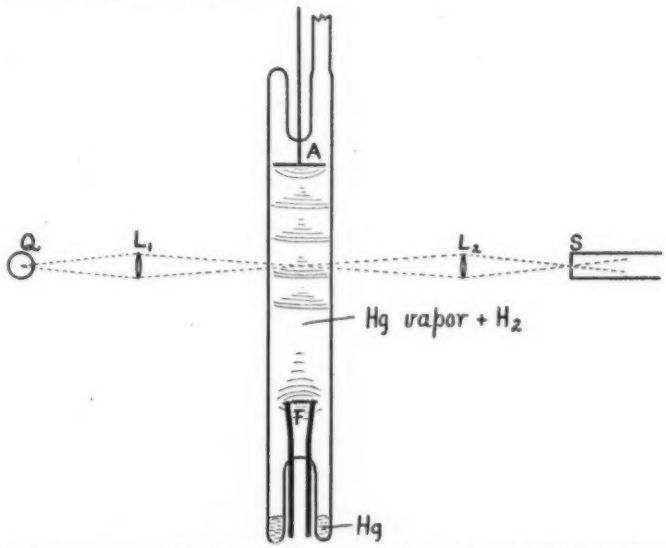

Fig. 3. Arrangement of apparatus for the study of the distribution of excited Hg atoms in various parts of the discharge. The concentration of excited atoms was taken to be proportional to the absorbing power for subordinate series lines.

In confirmation of the theory, it was found that the regions between striations did not detectably absorb these lines, the striations did absorb them, and the absorption was greatly increased if the amount of hydrogen was diminished.

A second experiment was devised to show that the action of the hydrogen was really to get rid of the excited atoms, and not something else. A discharge tube of the form shown in Fig. 4 was arranged horizontally and in it was placed a long glass boat MN containing tungsten oxide W_2O_3. This is a yellow powder which turns blue when reduced, and is known not to be reduced by molecular hydrogen at the temperatures involved in the present experiment. Langmuir[12] has shown that

[11] A complete report of various spectroscopic observations of the emission and absorption spectrum is to be published later.

[12] Langmuir, J. Am. Chem. Soc. **35**, 931 (1913)

this oxide is reduced by *atomic* hydrogen, and Cario and Franck[13] and Duffendack and Compton[14] have used such reduction to prove the dissociation of hydrogen which occurs in the presence of excited mercury atoms.

On the present theory, the excited atoms are produced in the luminous striations. Those which do not revert to the normal state through radiation may do so by expending their energy to effect the dissociation of hydrogen molecules with which they collide. We should therefore expect to find an excess of atomic hydrogen in the region of the luminous

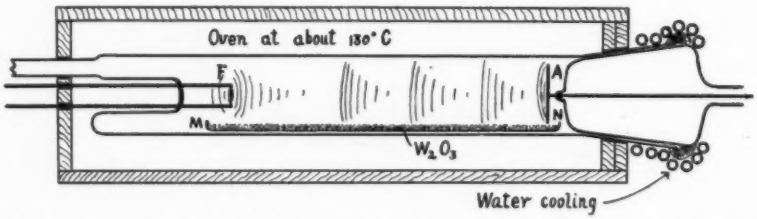

Fig. 4. Apparatus by which it was proved that hydrogen is dissociated most rapidly in the luminous regions of the striations. The atomic hydrogen was detected by its reduction of tungsten oxide, and its formation was ascribed to collisions of the second kind with excited mercury atoms.

striations and to detect this atomic hydrogen by its reducing action on the tungsten oxide. This was actually done. The blue patches which appeared on the surface of the oxide corresponded in position with the luminous striations. They were not as sharply defined as were the striations, owing, without doubt, to the diffusion of the atomic hydrogen away from its place of production. Rather intense discharges were necessary to give detectable reduction before the whole surface became rather uniformly reduced by atomic hydrogen from the region of the filament. Nevertheless the results were in conclusive support of the theory.

In this connection it may be noted that, although no hydrogen lines are found, the striations show strongly the band spectrum of HgH,[15] which is evidently associated with the action of excited mercury atoms on hydrogen, although it is not in the process of dissociation that the radiation from HgH occurs, since we failed entirely in an attempt to excite this spectrum by strongly illuminating with 2536 resonance radiation a mixture of mercury vapor and hydrogen. Apparently the HgH

[13] Cario and Franck, Zeits. f. Phys. **11**, 161 (1922)
[14] Duffendack and Compton, Phys. Rev. **23**, 583 (1924)
[15] Turner and Compton, Phil. Mag. **48**, 360, 1924.

molecules may be formed in the presence of excited atoms but require additional excitation to produce the band spectrum.

Finally, an experiment was tried to clinch the argument. According to the theory, an impurity is effective in producing striations only because it acts to decrease the concentration of excited atoms. Hydrogen does this because the energy of the excited mercury atoms (4.7 to 5.5 volts) is sufficient to dissociate hydrogen molecules, either at collisions of the second kind or by actual chemical oxidation. If helium were used in place of hydrogen, we should not expect it to have any effect on the excited atoms, since they would need about 20 volts energy to produce any change in a helium atom. So we mixed pure helium, in place of hydrogen, with the mercury vapor and found that the positive column remained unchanged and uniform for amounts of helium up to more than a thousand times the amount of hydrogen that would have been effective in causing striations. Still larger amounts of helium did produce a tendancy to striations, which may have been due to traces of impurity in the helium or to collisions of the second kind of the sort in which the energy of the excited atom is transformed into kinetic energy of the impacted molecule.

(2) Potential Distribution and Ion Concentration

To study potential distribution and ion concentration in the discharge the method recently devised by Langmuir[16] was employed, as being the most powerful one available and free from errors inherent in the older, exploring electrode method. The results of this study are being published in detail by one of us[17] and will therefore be discussed but briefly here. A short length (about 3 mm) of 4 mil tungsten wire, projecting from the tip of a fine glass tube, served as the exploring electrode and could be moved to any position between the hot filament cathode and the anode of the discharge tube. It was connected, through a multi-range millivoltmeter, to a potential divider by which its potential could be adjusted to any value relative to the anode or the cathode. When its potential V is negative with respect to that of the gas V_0 in the immediate vicinity, the negative current density to it is, approximately

$$I = I_- - I_+ \tag{4}$$

$$I_- = ne\sqrt{\frac{e\bar{V}}{3\pi m}}\,\epsilon^{-3(V_0-V)/2\bar{V}} \tag{5}$$

$$I_+ = Ne\sqrt{\frac{e\bar{V}_+}{3\pi M}} \tag{6}$$

[16] Langmuir, Science **58**, 290 (1923); Gen. Elec. Rev. p. 731 (1923)
[17] McCurdy, Phil. Mag. **48** (1924)

where I_- and I_+ are the currents of electrons and of positive ions, respectively, reaching the electrode, n and N are the numbers per unit volume of electrons and positive ions, m and M are their masses and $\bar{V}$ and $\bar{V}_+$ are their average energies in equivalent potential drop. Eqs. (5) and (6) assume Maxwellian distributions of velocities and Eq. (6) is subject to corrections involving the radius of the positive ion sheath which surrounds the electrode, for which the reader is referred to the more complete report of this phase of the work.

The experimental procedure is to determine I_+ by measuring I at some value of V so negative that I_- is negligible. This, with the correction noted above, gives the value of I_+ appropriate to any value of V less than V_0. Knowing I_+, it is added to any value of I to obtain the corresponding I_-. Then, if the logarithms of these values of I_- are plotted against the corresponding values of V, a straight line is obtained, for, from Eq. (5)

$$\log I_- = \log \left(ne \sqrt{\frac{e\bar{V}}{3\pi m}} \right) - \frac{3V_0}{2\bar{V}} + \frac{3V}{2\bar{V}} . \tag{7}$$

It is to be noted that this equation holds only for $V < V_0$. If $V > V_0$, the space charge around the electrode is negative instead of positive and the current is subject to a different law. Thus a "break" in the log I_--V curve indicates the point at which $V = V_0$ and therefore gives V_0, the potential of the gas. The slope of the straight line is $(3/2\bar{V})$ and therefore gives the average energy $\bar{V}$ of the electrons. The ordinate at $V = V_0$ is $\log (ne\sqrt{e\bar{V}/(3\pi m)})$ and enables the number n of electrons per unit volume to be calculated. Similarly, from Eq. (6), if the average energy $\bar{V}_+$ is estimated from mobility considerations, or otherwise, the concentration of positive ions can be calculated.

Fig. 5 gives four typical experimental log I_--V curves, taken at positions noted in the upper left hand corner, and also in Fig. 2. The circles in each case represent the point at which $I_- = I_+$, i.e. the point which would have been taken as the potential of the region by the old exploring electrode methods. The true potentials are given by the discontinuities, marked with arrows. That this interpretation of the discontinuities is correct was checked by a second method, also suggested by Langmuir, in which the exploring electrode was a short, 1 mil tungsten filament of hairpin form, which could be heated by closing a switch in a battery circuit. The true potential of the region is found by decreasing the potential of the filament from a positive value to the least potential at which the current to the filament is independent of its temperature, for, at any lower potential, thermionic emission occurs and at higher

potentials it is suppressed. The two methods gave identical results for the potential of the space.

Neglecting a small error at the bottom, amounting to only a few per cent of the total current and probably due to an approximation in making the correction noted above, Curves III and IV (Fig. 5) are straight lines showing that we have a Maxwellian distribution of velocities among the electrons in the regions of glow in the striations, at least for velocities below the critica velocity for inelastic impact. Curves I and II show that there is a departure from a Maxwellian distribution in the dark

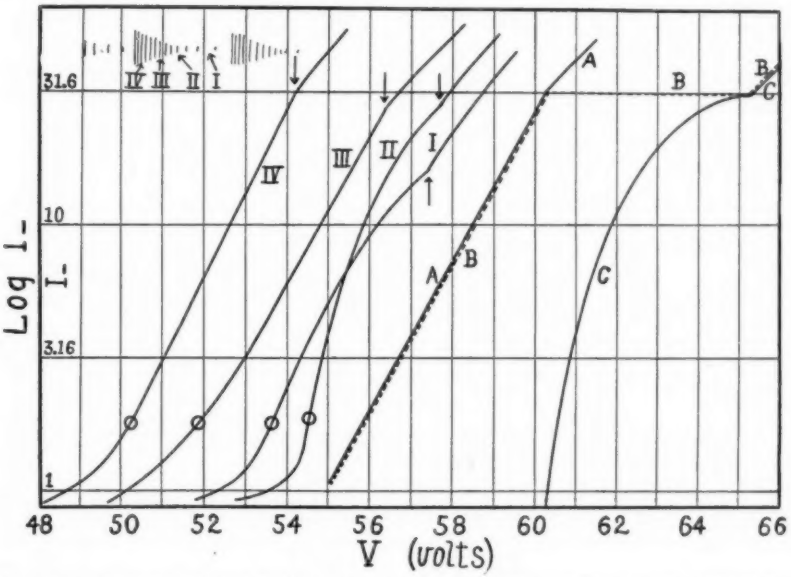

Fig. 5. Curves I, II, III and IV show the electron current to an exploring electrode placed in the parts of a striation noted in the upper left corner of the diagram. The arrows indicate the potentials of the region with respect to the cathode, and from the curves the mean kinetic energy and concentration of the electrons may be calculated. Curves A, B, C illustrate the cause of the departure of the experimental curves from straight lines in the dark spaces between striations.

spaces between striations. This departure is probably accounted for by the fact that the electrons which start with a Maxwellian distribution in the striations, fall through nearly equal potential differences before they collide inelastically in the next striation. If all fell through exactly the same potential difference, the electrons with the Maxwellian distribution shown by Curve A would reach the next striation with a distribution shown by Curve B. Owing to the fact that the electrons may collide inelastically whenever their speed exceeds the critical speed (about 5 volts) and that not all inelastic impacts occur exactly in the same region,

the actual distribution curve just in front of a striation would be expected to be suppressed at the higher velocities and rounded off like Curve C, which is the type of the observed curves in this region.

We are led thus to the conclusion, in agreement with our theory, that most of the inelastic collisions occur in the striations and that the energy requisite for inelastic impacts is gained in the potential drop between striations.

Furthermore, we are led to the interesting and significant conclusion that *a region of ionization gives a Maxwellian distribution of velocities to the electrons in this region*. The mechanism producing this distribution is not yet clear. It is known on dynamical grounds that such a distribution results from sufficiently numerous collisions, but this cannot be the complete explanation here, else there would be no reason for not anticipating a Maxwellian distribution also in the dark spaces between striations. This conclusion is also in agreement with observations of Langmuir and Jones,[18] who found Maxwellian distributions of velocities among electrons in a region of intense ionization where, however, the mean free path was so long in comparison with the dimensions of the apparatus that an explanation on the basis of momentum transfers at collisions is impossible.

It should be mentioned that our experiments showed Maxwellian distributions everywhere in the Faraday dark space and negative glow except within the region of the cathode drop, which we could not investigate on account of its small thickness.

The potential distribution in the discharge in one typical case is shown in Fig. 6, together with the electron and positive ion concentrations in various regions. It is seen that there is exact agreement with the predictions of the theory as to ion concentration, regions of positive and negative space charge and general shape of the potential distribution curve. Mechanical difficulties prevented the continuation of these curves clear up to the cathode and anode.

The potential drop between striations is very close to 5 volts, which indicates that the ionization occurring in the positive column is of the cumulative type. Evidently the majority of inelastic impacts result in the formation of excited atoms and only the relatively small number of inelastic impacts with previously excited atoms may result in ionization.

Table I gives numerical results of one set of measurements, with calculated values of electron energies and of electron and ion concentrations. The calculations of positive ion concentrations were made from

[18] Langmuir and Jones, Science **59**, 380 (1924)

Eq. (6), using values of the mean energy of the positive ions computed from the ordinary mobility equation, with the aid of the observed values of the potential gradient. This calculation involves considerable uncertainty, which probably accounts in part for the apparent excess in the number of electrons over positive ions in the Faraday dark space, an

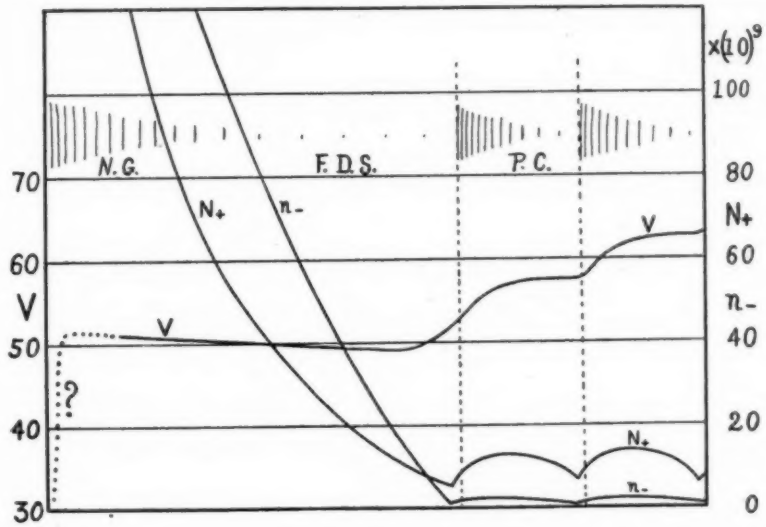

Fig. 6. Experimental values of electron concentration n, positive ion concentration N and potential V in various parts of the discharge. The values of N are rough approximations, but the values of n and V should be quite accurate.

excess much larger than is possible to reconcile with Poisson's equation. Yet the order of magnitude of the values should be correct. The values for the electron concentrations involve no such uncertainty, and should be quite reliable.

TABLE I

V_0 is the potential of the gas with respect to the cathode;
$\overline{V}$ is the mean energy of the electrons, in volts;
n and N are the concentrations of electrons and positive ions, respectively.

Position	V_0	$\overline{V}$	n	N
Between striations	57.4	1.2	0.38 (10)⁹	10.0 (10)⁹
Anode side of striation	57.3	1.5	0.75	11.8
Middle of striation	56.6	1.4	1.30	12.4
Cathode side of striation	54.2	1.7	1.55	11.0
1.3 mm. into Faraday dark space	51.7	1.5	0.65	5.6
13 mm into Faraday dark space	49.2	0.40	21.0	15.5
20 mm into Faraday dark space	49.6	0.24	53.5	26.2
Anode side of negative glow	50.0	0.40	101	51.2
Middle of negative glow	50.1	0.91	130	900

(3) Existence of Negative Atom-ions

The concentration of positive ions N exceeds that of electrons n in the positive column by amounts which seem to be too large to be accounted for by an uncertainty in the manner of calculation. It is easily shown that this apparent difference is enormously larger than is allowable by Poisson's equation. The only reasonable explanation seems to be that there exist negative ions of atomic mass in concentration comparable with that of the positive ions. Such ions would not have been detected in the present experiments, since they would carry but an insignificant portion of the negative current, owing to their large mass relative to that of the electrons. In support of this hypothesis it may be mentioned that several other phenomena have pointed to the same conclusion, and that Franck and Grotrian[10] have shown theoretical reasons for expecting excited atoms to possess electron affinity and to form fairly stable negative ions. A direct experimental search for such negative ions of monatomic atoms is being undertaken.

(4) Mechanism of Light Production

These experiments prove, incidentally, that the greater part of the light produced in the glow discharge comes, *not from recombination* of ions and electrons, *but from excitation* of neutral atoms by electron impacts. If light were produced only by recombination of ions and electrons, we should expect the light from the negative glow to be several thousand times as intense as that from the positive column, since the rate of recombination is proportional to the product of the concentrations. On the contrary, the positive column is actually observed to be brighter than the negative glow. It seems safe to conclude, therefore, that practically all the light from the positive column comes in the process of readjustments within the atoms following excitation, or partial ionization. The different character of the light from the negative glow is probably due to the contribution of light from ion recombination in this region, giving a different distribution of energy in the spectrum.

(5) Relation of Glow Discharge to Arc

On the basis of the foregoing analysis of the conditions in a glow discharge from a hot cathode, there is no reason for drawing a sharp distinction between the glow discharge and the arc from a hot cathode. The distinction seems to be purely a matter of distance between the electrodes. If this distance is so small that the ions produced just beyond the cathode drop diffuse toward the anode, recombining on the way, and are still

present in large concentration at the anode, then the discharge shows no positive column, and is of the arc type. In other words, if, in Fig. 2, the anode were placed to the left of the point *f*, the conditions between it and the cathode would be the same as shown except for secondary local effects due to the presence of the electrode, and we should have an arc discharge. If the anode were to be placed at the point *e*, we should have an arc at a minimum voltage, which might be less than the ionizing potential of the gas. It seems that the two types of discharge are really subject to the same explanation, the arc being simply the negative glow of the longer glow discharge.

This analysis suggests numerous possibilities of further test and of more quantitative treatment in particular regions of the discharge, where the experiments show that some of the factors governing the current flow become either negligible or of preponderating importance.

We wish to thank Mr. C. C. Van Voorhis for his able assistance in carrying out some of the experiments.

PALMER PHYSICAL LABORATORY,
PRINCETON UNIVERSITY,
August 15, 1924.

ABSORPTION OF CATHODE RAYS IN ALUMINUM FOIL

H. M. TERRILL

ABSTRACT

The experimental method used is a modification of that of Whiddington and of Schonland. Homogeneous rays of accurately known velocity are provided by fitting a Coolidge cathode to a high vacuum tube and exciting it by high tension direct current. The stream of electrons traverses the foil and enters a Faraday cylinder connected to a galvanometer. The foil may be swung aside to permit measurements of the direct beam. One to three foils of rolled aluminium .00031 cm thick were studied with voltages 18 to 52 kv. Up to the vicinity of the range limit, Lenard's equation, $I/I_0=e^{-\alpha x}$ is found to hold, where x is the thickness. Within this range of velocities, 8.09 to 12.23×10^9 cm/sec., the values of α satisfy the approximate relation derived by J. J. Thomson, $\alpha v^4=.268$ (10^{44}), where v is the mean of the velocities before and after penetrating the foil.

PURPOSE OF THE INVESTIGATION

MEASUREMENTS of the absorption of cathode rays may be divided into two classes; first, measurements in which non-homogeneous rays are used, and second, those in which rays of uniform velocity are employed. In the first group may be placed the work of Lenard[1] and his followers, done largely with filtered rays. In the second group, Whiddington[2] and later Schonland[3] worked with rays of uniform velocity produced by magnetic sorting, but the results obtained by them are not in agreement with each other nor with those of the earlier writers.

It is believed that the lack of agreement in these results may be traced to the difficulties of velocity determination. Since the absorption varies rapidly with the velocity, this must be known accurately. Even in the work done with homogeneous rays, the actual velocities employed were known by indirect measurements only.

The purpose of the present work is to repeat these measurements, using homogeneous rays of accurately known velocity. These can be secured by using a high vacuum tube fitted with a Coolidge cathode and exciting the tube with constant potential direct current. The velocity, or corresponding voltage, is then susceptible to direct measurement.

In experiments here reported, the voltage was measured by an electrostatic voltmeter calibrated by means of a milliameter and a metallic high

[1] Lenard, Quantitatives über Kathodenstrahlen, Heidelberg, 1918.
[2] Whiddington, Proc. Roy. Soc., **89**, 559 (1914)
[3] Schonland, Proc. Roy. Soc., **104**, 235 (1923)

resistance multiplier. This multiplier was wound with manganin wire and was immersed in a bath of transformer oil. It had a total resistance of 19.413 megohms.

Apparatus and Method

A diagram of the tube is given in Fig. 1. It is built along the general lines of a Coolidge x-ray tube, save that the cathode end is closed with a ground joint *S*, and that the anode is hollow, a portion of the cathode rays entering through a small hole in its face.

The cathode is mounted on a stem attached to the ground stopper, and is thus convenient to get at and to adjust.

The anode *A* is made of copper, and is in two parts, the upper being removable. The face of the target struck by the rays is protected by a thin disk of molybdenum *M* standing on short pins, a few millimeters away from the surface, and having a thin-edged hole at its center through which the cathode rays enter. Those electrons which do not pass through the hole dissipate their energy in the molybdenum, and this, running at a higher temperature than the rest of the anode, radiates heat more effectively, insuring a low temperature for the interior.

The foil to be examined is clamped in an iron frame *F*, which may be swung aside by a magnet held outside the tube, thus permitting measurement of the direct beam. At the bottom of the anode the electrons enter a Faraday cylinder *C*, held on silica insulators. This cylinder has a converging mouth, and is carefully adjusted to stand within a fraction of a millimeter of the foil. The opening is larger than that through which the electrons enter the target; the large mouth, held close to the foil, serves to catch those electrons which have penetrated the foil but have been scattered at large angles to the entering direction.

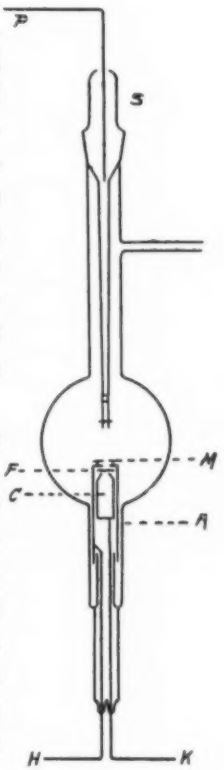

Fig. 1. Diagram of the tube.

Sealed through the glass at the lower end of the tube are two wires *H* and *K*. On the first, the current from the outer face of the target runs to ground through a milliameter, and on the second, that from the Faraday cylinder is grounded through a universal shunt and galvanometer. In the galvanometer, uniformity of scale is more important than high sensi-

tivity, and the uniformity was checked over the length of the scale by means of a standard cell and resistances. The sensitivity averaged about 600 mm per microampere on the scale used. The whole galvanometer circuit is enclosed in a grounded metallic shield.

It is necessary that the secondary electrons from the foil itself be excluded from the Faraday cylinder. The majority of these have a velocity of less than ten volts, and they are practically all stopped by thirty volts, this point having been carefully tested by Schonland over a range of initial velocities about the same as that used here. In order to stop these secondary electrons, the Faraday cylinder is kept at -100 volts with respect to the foil by a battery of dry cells in the galvanometer circuit, the field being equalized over the foil by a light grid within the mouth of the cylinder.

The arrangement for pumping and baking out the tube, and the supply of high tension direct current for exciting it, are the same as described in a previous article.[4]

Rolled foil was used, the thickness being determined by weighing a measured area. Runs made with pieces cut from different parts of the same sheet agree within the limits of experimental error, showing the thickness to be uniform.

In making observations, the voltage is set at some desired point, and a series of readings of the galvanometer taken with the foil alternately in and out of the beam of rays.

A sample set follows:

Voltage 33.9 kv Absorber .00031 cm

Galvanometer deflections

Absorber in	Absorber out
10.9 cm	24.7 cm
10.9 cm	24.6 cm
10.8 cm	24.2 cm

$I/I_0 = 10.86/24.5 = .443$

Data and Results

Values of I/I_0 were obtained for three different thicknesses of aluminum, .00031, .00062 and .00093 cm for voltages up to 50 kv. Fig. 2 shows the variation of I/I_0 with the voltage for the different thicknesses.

The fundamental equation for I/I_0, usually known as the law of Lenard, is

$$I/I_0 = e^{-ax} \qquad (1)$$

where x is the thickness and a depends on the density of the material and the velocity of the rays. For any voltage, given the fraction transmitted

[4] Terrill, Phys. Rev. 22, 101 (1923)

through the foil or I/I_0, it is possible to substitute in (1) and obtain a value for a. Since a is a function of the velocity, allowance must be made for the velocity lost by the rays in traversing the foil.

Accurate measurements of the loss of velocity have been recently made by the writer.[4] Within the limits of experimental error, it was found that the approximate formula of J. J. Thomson held. In voltage form this equation is

$$V_0^2 - V_x^2 = bx \tag{2}$$

and for aluminum, $b = 1.1 \times 10^{12}$.

Additional confirmation of this equation is obtained from the present results. The value of V_0 found by placing $V_x = 0$ in Eq. (2), gives the voltage corresponding to the so-called range limit at which the transmitted fraction should equal zero. Values of V_0 for the three thicknesses come out 18.4, 26.1 and 32.0 kv, which is in satisfactory agreement with the points at which the I/I_0 curves meet the axis, the peculiar shape of the curves preventing an exact determination of these points.

Returning to Eq. (1), corrections for loss of velocity may now be made. It is evident that by applying this equation to measurements made with a foil of finite thickness, a value of a is obtained that corresponds, not to the initial velocity v_0, but to an average of the changing values assumed by the velocity as the foil was traversed. The value of this average will be called the effective velocity. It may be taken as approximately equal to $\frac{1}{2}(v_0 + v_x)$ for thin foils such as used here.

Values of I/I_0 for different thicknesses, but corresponding to the same effective velocity may now be substituted in Eq. (1), and if this equation is to hold, the value of a should come out the same for all thicknesses at any given velocity. This is found to be the case as long as the values of I/I_0 are not taken too close to the range limit, but as that point is approached discrepancies appear.

Referring to the curves in Fig. 2, it will be observed that at the lower end, as the range limit is approached, they change curvature and tail off to the zero axis. Here the beam has become more diffuse, being composed of electrons that have suffered wide angle scattering, and a gradual deviation from Lenard's law is not surprising, especially since at the limit, the approximation used for the effective velocity may not hold.

The values of I/I_0 above the points marked A on each curve closely satisfy Lenard's equation. Values of a obtained from these are given in Table I. For each value of a is listed the corresponding velocity and its voltage equivalent. For convenience of reference the fourth column gives the values of a divided by the density of aluminum.

These values of a are considerably lower than those given by Lenard, which are averaged from the work of most of the earlier writers. Whiddington gives no results for aluminum above 11.5, so a comparison with his results cannot be made. Schonland does not give values of a, but computation from his values of I/I_0 leads to values of a lower than given here.

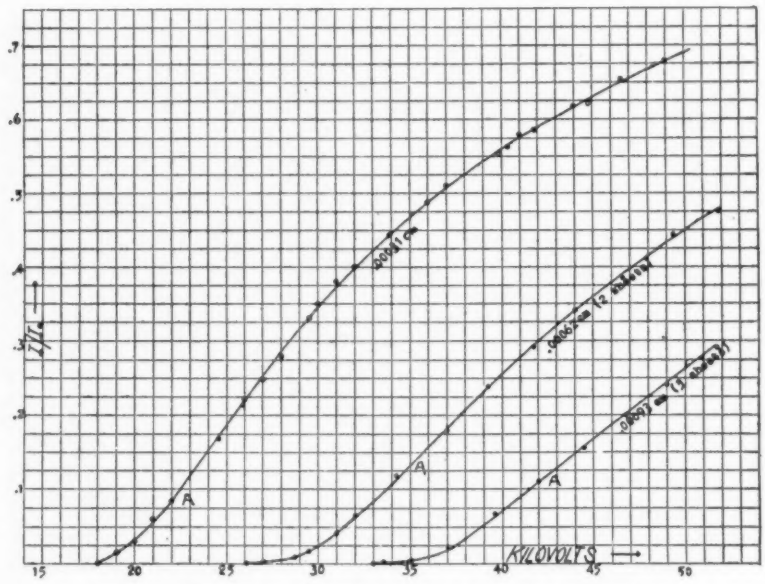

Fig. 2. Transmitted fraction as function of initial voltage.

The last column lists the values of v^4a which come out practically constant, so that, within the range of velocities given, the values of a are very closely proportional to the inverse fourth power of the velocity. This is in agreement with an approximate relation derived by J. J. Thomson[5] expressing a as the product of a constant times v^{-4} and a logarithmic factor whose variation may be neglected over a narrow range of velocities.

Taking the mean value of the constant from the last column of Table I we may write $av^4 = .268\ (10^{44})$.

Lenard does not give any law of variation of a with the velocity, although some of his earlier results were cited by Thomson in support of the above equation. Whiddington found av^4 constant for the case of gold, but for aluminum and other metals he obtained a more complicated

[5] J. J. Thomson, Conduction through Gases, p. 377.

relation. Seitz,[6] whose results are not included in Lenard's averages, found α proportional to v^{-6}.

Work is in progress to extend the present results. It is hoped to run at higher voltages, thus permitting measurements with thicker foils and

TABLE I

Velocity (10^9cm/sec)	Voltage (kv)	α (10^3)	$\alpha/2.7$ (10^3)	αv^4 (10^{44})
8.09	19.8	6.17	2.285	.264
8.55	22.2	4.90	1.815	.262
8.96	24.5	4.10	1.518	.264
9.29	26.4	3.48	1.289	.259
9.74	29.1	2.95	1.093	.265
10.07	31.3	2.59	.960	.262
10.39	33.5	2.31	.856	.268
10.64	35.3	2.12	.786	.271
10.94	37.6	1.91	.708	.273
11.22	39.8	1.74	.645	.276
11.48	41.9	1.56	.578	.271
11.74	44.1	1.43	.530	.271
11.99	46.2	1.31	.485	.270
12.23	48.3	1.21	.449	.271

metals of higher density. Further, a modification of the present apparatus is under construction by means of which may be determined not only the fraction of the beam penetrating the foil but at the same time the velocity distribution of this transmitted fraction.

INSTITUTE OF CANCER RESEARCH,
COLUMBIA UNIVERSITY,
September 8, 1924.

[6] Seitz, Ann. der Phys. **12**, 860 (1903)

THE MOBILITY OF ACTINIUM ACTIVE DEPOSIT IONS

By Henry A. Erikson

Abstract

The mobility of the ions in actinium active deposit is determined by drawing them across an air stream having a speed of 20 m/sec., by means of an electric field of over 100 v/cm, and allowing them to deposit on a metal plate. The amount of deposit at various distances along the plate is determined by the ionization it produces through a slit in the wall of an ionization chamber. It is found that two positive ions are present, one of which has a mobility of about 4.35 cm/sec/volt/cm and the other 1.55. The first is greater than that of monomolecular ions of air and the second greater than that of bi-molecular ions of air (Phys. Rev. **24,** 1924). The interpretation advanced is that the more rapid ions are atomic actinium A and B, and that the slower ion is formed when the more rapid ion attaches itself to a neutral molecule (presumably air). A comparison of the results in the case of these heavy ions with those in the case of the air and CO_2 ions shows that the mobility of an ion is practically independent of its mass.

THE mobility in air of the ions composing the thorium active deposit was investigated by Rutherford[1] and by Franck.[2] The mobility values found were approximately the same as for the positive air ion.

It became of interest to determine the mobility of the active deposit of actinium by means of a method having a resolving power such as would lead to the detection of different ions if present.

The method is essentially that used to study the initial ions of air and CO_2.[3] It may be briefly described as follows.

A dish containing an actinium salt was placed at D inside a tube T (Fig. 1). Air containing the actinium emanation and its disintegration products was drawn into the space between two plates A and B by means of a fan H. The velocity of the air between the plates A and B was of the order of 2000 cm/sec. The air containing the active deposit entered at E through an opening $.25 \times 5$ cm. The rest of the air passing between the plates A and B was taken from the room and was free from emanation. The plate A was kept at a positive potential by means of the battery G; plate B was grounded. To secure a uniform field, wires whose potential varied in steps were placed at the sides of the space between A and B. Since the active deposit is positively charged it is repelled to the plate

[1] Rutherford, Phil. Mag. **95,** (1903)
[2] Franck, Ber. Deutsch. Phys. Ges. **7,** 397 (1909)
[3] Erikson, Phys. Rev. **24,** 502 (1924); **20,** 117 (1922)

B across the air stream and is deposited on B. If particles differing in mobility are present they will be deposited at different points on B. After exposure of plate B for a given time it was removed and placed beneath an ionization chamber in the bottom of which there was a slit at right angles to the plate as shown in Fig. 2. The relative amounts of active deposit at different points on B was determined by measuring the current produced in the ionization chamber A.

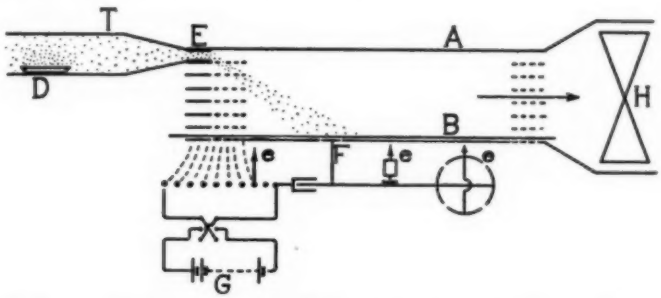

Fig. 1. Apparatus for getting ionic spectrum of actinium deposits.

The results obtained after an exposure of 30 minutes with 4500 volts between the plates A and B, Fig. 1, are given in Fig. 3.

The current obtained in the ionization chamber, Fig. 2 is plotted as ordinates against the down stream distances as abscissas. The order in which the readings of the upper curve were obtained was from right to left. The lower curve was obtained by immediately reversing the order in which the readings were taken. The diminished ordinates are due to the decay of the active deposit.

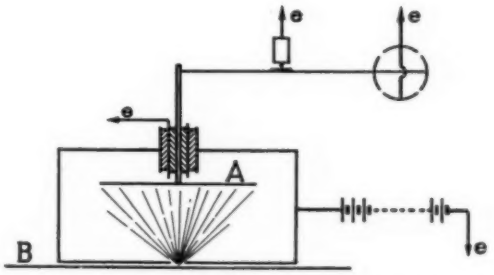

Fig. 2. Apparatus for determining activity on plate B (Fig. 1) as function of distance from one end.

It thus becomes evident from the two maxima that there are two active bodies present, both positively charged. The first maximum corresponds to a mobility of about 4.35 cm/sec/volt/cm and the second

to a mobility of about 1.55. The average mobility ratio found for these two active bodies is 2.80.

As to the nature of the two bodies no definite conclusion can as yet be advanced. The results however suggest the following discussion.

Actinium emanation has a half value period of 3.9 sec. and is an electrically neutral gas in the atomic form. The emanation atom disintegrates into actinium A which has a half value period of .002 sec. and is positively charged. The actinium A which results from the emanation which enters as such at E gives a continuous spectrum on plate B, Fig. 1. The actinium A which enters at E is due to the emanation which disintegrates in a distance of the order of 2 cm before E, as the air velocity towards the E end of the funnel is about 2000 cm/sec. The swifter of the two bodies passes from E to plate B in about .002 sec. It is thus seen that

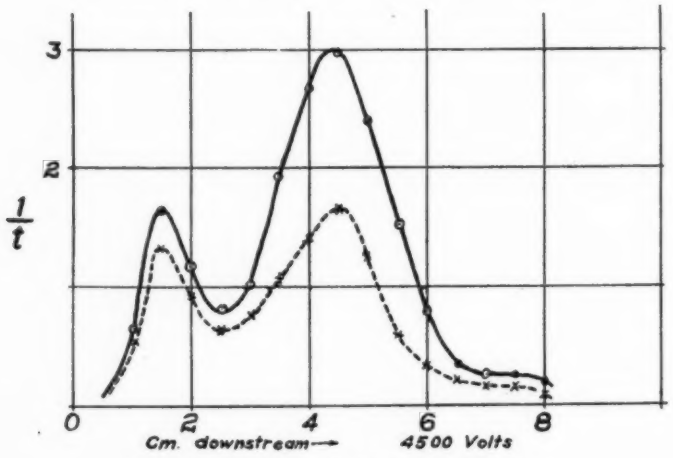

Fig. 3. Spectrum of actinium depositions.

it is possible for the actinium A which leaves E to reach plate B and that the chances are that it is in the atomic form, as owing to the short interval of time it is not able to unite with other molecules to any appreciable extent. The actinium A which changes into actinium B after leaving E will give a continuous spectrum which is denser at the down stream end. The positive actinium B which enters at E and which is also in the atomic form will come down with the atomic actinium A as the effect due to the difference in mass may be neglected. The first maximum therefore on this view is due to both atomic actinium A and B. As actinium B has a half value period of 36.3 min. it remains as such in the air for a considerable time and has a good chance of uniting with an air molecule so as to form a positive ion three atoms large. On this view the slower of the two

active bodies is a positive ion three atoms large and as stated above corresponds to a mobility of about 1.55.

In Fig. 4, curve B is for the positive air ions and is inserted for comparison. This curve was obtained by ionizing the air entering at E by means of α rays from polonium and by measuring the current to the strip F, Fig. 1, as a function of its position.[4] The maximum C is for the initial positive air ion and maximum D is for the final positive air ion. It is thus seen that the maximum for the slower active body comes between the initial and final air ions. In a study of the air ions,[3] the conclusion was reached that the negative and initial positive ions are each two atoms large and the final positive ion is four atoms large. If the slower active body found above is three atoms large, it should have, as the results show, a mobility value lying between the mobility of the two positive air ions,

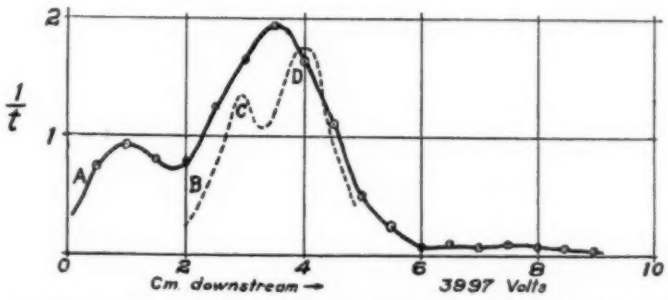

Fig. 4. Curve A, spectrum of actinium ions; curve B, spectrum of air ions, under the same conditions.

provided it has not a characteristic molecular structure[5]. That the first maximum is due to atomic actinium A and B and that the second maximum is due to actinium B and possibly A, after a union resulting in an ion three atoms large is therefore, it would seem, a probable conclusion.

It has not been found possible to show the transition with time of one of the active bodies into the other although the transition can quite easily be shown in the case of the initial positive air ion. The variation due to the decay of the emanation and recombination due to the heavy ionization interpose difficulties which have not as yet been overcome.

It is of interest to note that although these two active bodies have masses of the order of 15 times the masses of the air ions their mobilities

[4] See preceding paper, l.c.[3]

[5] Results obtained in the case of different ions seem to indicate a difference, for example, between a three atom ion formed through the loss of an electron by a three atom molecule and the three atom ion formed by the attachment of a positive atom to a neutral two atom molecule.

are of the same order, the swifter moving more rapidly than either of the air ions. This shows that the mass of the ion is not an appreciable factor. The mobility is determined chiefly by the charge, the structure of the ion and the nature of the medium through which it moves.

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UNIVERSITY OF MINNESOTA,
May 10, 1924.[6]

[6] Received July 21, 1924.

PROOF OF THE INVARIANCE UNDER THE LORENTZ TRANSFORMATION OF THE EQUATION OF MOTION OF THE ELECTRON

By Leigh Page

Abstract

The equation of motion of the electron relative to an inertial system S' in which it is momentarily at rest is

$$e\mathbf{E}'=m\mathbf{f}'-n\dot{\mathbf{f}}'\ \ldots\ +a_n\mathbf{f}'^{(n)}\ \ldots \qquad (1)$$

where $\mathbf{E}'$ is the electric intensity, $\mathbf{f}'$ the acceleration, etc. To obtain the equation of motion relative to a system S with respect to which the electron has a velocity $\mathbf{V}$, it is necessary to substitute for the components of $\mathbf{E}'$, $\mathbf{f}'$, etc. their values in terms of the components of $\mathbf{E}$, $\mathbf{H}$, $\mathbf{V}$, $\mathbf{f}$, etc. measured in S as specified by the usual transformations. In this paper it is shown that the equation of motion thus obtained is invariant under the Lorentz transformation. The method of proof consists in writing down an equation, all the terms of which are shown to be four-dimensional-vectors, which reduces to (1) when referred to the system S' in which the instantaneous velocity $\mathbf{V}'$ of the electron is zero.

THE equation of motion of the electron is deduced from the hypothesis of Lorentz that the force on the electron due to an impressed field is at every instant balanced by the force exerted on this particle by its own field. This hypothesis explains mass on purely electromagnetic grounds, exhibiting Newton's second law of motion as a deduction from electromagnetic theory. This achievement is perhaps the greatest which has been attained by the electron theory.

However, the equation of motion of the electron contains in addition to the mass reaction $m\mathbf{f}$, terms involving the rate of change of the acceleration $\mathbf{f}$ and other higher time derivatives. While it is well recognized that if the terms involving the derivatives of the acceleration are discarded,[1] the incomplete equation of motion so obtained is invariant under the Lorentz transformation, the impression seems to be somewhat general that the complete equation may not satisfy the condition of invariance. Therefore it has seemed worth while to give the following formal proof of the invariance of the complete equation.

Consider an electron some point of which—for example, the center—has any arbitrarily assigned motion. At any instant this point will be momentarily at rest in some one inertial system S'. Relative to S' the electron will be supposed to be, at this instant, a uniformly charged

[1] E. Cunningham, Relativity and the Electron Theory, pp. 75 and 79.

spherical shell of radius a, every point of which is momentarily at rest. Thus the motion of every point of the electron is completely defined in terms of the motion of any one point, such as its center.

Let $\mathbf{E}'$ be the electric field, as measured in S', which is impressed on the electron from outside sources, and $\mathbf{E}_1'$ the electric field due to the electron itself. Then if de is an element of the electron's charge, the Lorentz hypothesis requires that

$$\int \mathbf{E}' de = -\int \mathbf{E}_1' de$$

integrated over the surface of the electron. As $\mathbf{E}'$ may generally be considered to be sensibly constant over the surface of the electron, the left hand side of this equation becomes $e\mathbf{E}'$. The right hand side yields a series in the acceleration $\mathbf{f}$ and its time derivatives,[2] such that

$$e\mathbf{E}' = m\mathbf{f}' - n\dot{\mathbf{f}}' \cdots + a_n \mathbf{f}'^{(n)} \cdots \tag{1}$$

where m is the rest mass $e^2/6\pi ac^2$, n is $e^2/6\pi c^3$, and in general the coefficient a_n is a constant involving the charge and the radius a of the electron, and the velocity of light c.

To obtain the equation of motion relative to a system S with respect to which the electron has a velocity $\mathbf{V}$, it is necessary to substitute for the components of $\mathbf{E}'$, $\mathbf{f}'$, $\dot{\mathbf{f}}'$ etc. their values in terms of the components of $\mathbf{E}$, $\mathbf{H}$, $\mathbf{V}$, $\mathbf{f}$, $\dot{\mathbf{f}}$ etc. measured in S as specified by the usual transformations.[3] It has always seemed clear to the author that this process insured the invariance of the general equation of motion thus obtained. It will now be proved that such is the case. The method of proof consists in writing down an equation all the terms of which are shown to be four-vectors which reduces to (1) when referred to the system S' in which the instantaneous velocity $\mathbf{V}'$ of the electron is zero. Since all the terms of the equation are four-vectors, the three space components of this equation are equivalent to the equations obtained by making the substitutions mentioned above in the three components of (1). Hence this equation is the equation of motion referred to a system S relative to which the electron has any assigned velocity $\mathbf{V}$. But as it is a four-vector equation it is invariant under the Lorentz transformation.

Put $$K \equiv (1 - V^2/c^2)^{-\frac{1}{2}} \; ; \quad K' = (1 - V'^2/c^2)^{-\frac{1}{2}}$$

and denote by brackets that $\mathbf{V}'$ has been put equal to zero in the quantity enclosed therein. Put

$$\mathbf{R} \equiv \frac{K\mathbf{V} + \mathbf{k}_4 icK}{K'} \equiv \frac{\mathbf{P}}{K'}$$

[2] Page, Phys. Rev. **11**, 398 (1918); Introduction to Electrodynamics p. 52.
[3] Page, Introduction to Electrodynamics, pp. 16-18, 24 and 53.

where $\mathbf{k}_4$ is a unit vector along the time axis in S. Now $\mathbf{P}$ is known to be a four-vector, and hence

$$\mathbf{P}=K'\mathbf{R}=K\mathbf{V}+\mathbf{k}_4 icK=K'\mathbf{V}'+\mathbf{k}_4' icK' ,$$

and

$$\mathbf{R}=\mathbf{V}'+\mathbf{k}_4' ic . \tag{3}$$

Then the equation under consideration is

$$eK\left\{ \mathbf{E}+\frac{1}{c}\mathbf{V}\times\mathbf{H} \right\}+\mathbf{k}_4 i\frac{e}{c}K\mathbf{V}\cdot\mathbf{E}$$
$$=m\left[\frac{d\mathbf{R}}{dt'}\right]-n\left[\frac{d^2\mathbf{R}}{dt'^2}\right]\cdots+a_n\left[\frac{d^n\mathbf{R}}{dt'^n}\right]\cdots \tag{4}$$

The left hand side of this equation is a well known four-vector which transforms into the left hand side of (1) when referred to the system S' in which the electron is momentarily at rest. Furthermore it is clear from (3) that the right hand side of (4) is identical with the right hand side of (1) when referred to the system S'. Therefore it only remains to prove that the right hand side of (4) is a four-vector.

It will now be shown that each term in the right hand member of (4) is a four-vector. The proof rests upon the well-known fact that the differential operator

$$K'\frac{d}{dt'}=K\frac{d}{dt} \tag{5}$$

when applied to a four-vector yields a four-vector. Applying this operation to the four-vector

$$K'\mathbf{R}=\mathbf{P} \tag{6}$$

we have

$$K'^2\frac{d\mathbf{R}}{dt'}+K'\frac{dK'}{dt'}\mathbf{R}=K\frac{d\mathbf{P}}{dt} ,$$

and as

$$\frac{dK'}{dt'}=\frac{\mathbf{V}'\cdot\mathbf{f}'}{c^2}\Big/\left(1-\frac{V'^2}{c^2}\right)^{3/2} ,$$

it follows that

$$[dK'/dt']=0 , \quad [K']=1 ,$$

and therefore

$$[d\mathbf{R}/dt']=Kd\mathbf{P}/dt ,$$

which is known to be a four-vector. Repeating this process of differentiation $n-1$ times we obtain on the left hand side of the equation terms in $\mathbf{R}$ and its first n derivatives having for coefficients products of powers of K' and the first n derivatives of K'. Now the derivatives of K', as is

clear from the form of this expression, will consist of terms involving the powers of K' and factors of the form

$$\mathbf{V}'^{(\alpha)} \cdot \mathbf{V}'^{(\beta)} \tag{7}$$

where $\alpha+\beta \leq n$. If $\alpha=n$, $\beta=0$, and the factor vanishes when $\mathbf{V}'$ is put equal to zero. Therefore the only terms surviving are those for which α and β are each less than n. Now (3) shows that

$$[\mathbf{V}'^{(\alpha)}] = [d^\alpha \mathbf{R}/dt'^\alpha] .$$

So, provided $[d^\alpha \mathbf{R}/dt'^\alpha]$ and $[d^\beta \mathbf{R}/dt'^\beta]$ are four-vectors, (7) is a scalar invariant. Now let it be assumed that these derivatives are four-vectors for all α's and β's less than n. Then in the equation obtained by operating on (6) n times with the operator (5) and then putting $\mathbf{V}'$ equal to zero, every term is known to be a four-vector with the exception of $[d^n \mathbf{R}/dt'^n]$. Therefore this term must be a four-vector. Hence by a process of mathematical induction it may be proved that every term on the right hand side of (4) is a four-vector. But (4) represents the equation of the electron deduced from the hypothesis of Lorentz. Therefore this equation of motion is an invariant of the Lorentz transformation.

As a check on the above reasoning, the author has shown by actually carrying out the transformation that the second term on the right hand side of (4) transforms as a four-vector.

UNIVERSITY OF CHICAGO,
July 16, 1924.

THE PHOTO-ELECTRIC RESPONSE OF POTASSIUM AT LOW TEMPERATURES

By J. W. Hornbeck

Abstract

Variation of photo-electric currents from potassium with temperature, 20° to −180°C, for the visible spectrum.—Carefully purified potassium in a highly evacuated tube surrounded by a double walled container for liquid air, insulated by wool, was illuminated with monochromatic light of constant intensity. As independently observed by Ives, the current wave-length curve at −180°C is shifted to shorter wave-lengths with respect to the curve at 20°C, the shift decreasing from about 500 A at the long wave limit 6500 A, to 80 A in the blue. These shifts are reversible and reproducible. Monochromatic heating and cooling curves run approximately parallel to the temperature axis except between −80° and −100°, where they show a sudden change. This region is where a change of crystal structure is known to take place. The shifts observed, then, seem to be due in part, at least, to the change of crystal structure. This point needs further study. Slight sudden changes of temperature produced irregular effects which are not understood; they may possibly be associated with the condensation of new vapor or with cracks in the surface.

Introduction

THE independence of photo-electric properties and the temperature of the illuminated surface has been generally regarded as well established. This conclusion rests upon a large amount of experimental data reported by several men, working under different conditions and covering various temperature ranges.

However, a study of the experimental work on photo-electricity revealed noteworthy gaps in the evidence. Practically no work at all had been done at low temperatures. Only one investigation[1] was found which reached down to the temperature of liquid air; and the alkali metals had not been studied below room temperature. Dember,[2] in 1907, investigated sodium and potassium from room temperature up through their melting points, finding no consistent changes in the photo-electric currents with increasing temperature, but he did not use monochromatic light and he made no attempt to determine long-wave limits.

For these reasons and further because the alkali metals are apt to be extreme in their behavior, the study of pure potassium was undertaken down to the temperature of liquid air. The problem of potassium at low

[1] A. Lienhop, Ann. der Phys. **21,** 281 (1906)
[2] Dember, Ann. der Phys. **23,** 957 (1907)

temperatures had an additional point of interest due to the fact that Bidwell[3] recently discovered a sudden change in the thermo-electric power and also in the resistance of potassium at a temperature near $-110°C$, where a change in crystal structure occurs. Not knowing of the work of Ives[4] on this same problem, published in April, 1924, a number of the interesting effects which he reports were independently observed by the writer before Ives' paper appeared. These included the following: The absence of a well-defined long-wave limit for very pure potassium in a high vacuum; a lateral shift toward the short wave-lengths, of the curve in which photo-electric current per unit light intensity is plotted against wave-length, on cooling the cell to the temperature of liquid air; a corresponding shift in the long-wave limit; and the perfect reversibility of these phenomena.

After the appearance of Ives' paper, a detailed study of these effects was made for gradually rising temperature from $-180°C$ up to $+20°C$, and indications were obtained that the shifts of the curves are due, in part at least, to the change in crystal structure.

The Cell

The photo-electric cell, made of Pyrex, had electrodes of heavy tungsten wire. The receiving electrode ended in a circle of tungsten wire, crossed by two small wires at right angles to each other. After baking the glass system thoroughly in an electric oven at 360°C, the potassium was twice distilled and then poured over into the cell with the diffusion pump running continuously.

This work of pumping and filling the cell was done at the Case Research Laboratory, Auburn, N. Y., for which thanks are due to Mr. T. W. Case, particularly for expert assistance furnished by him. The diffusion pump was of the highly-efficient, all-metal type and, with the exception of the connection to the pump, no ground glass joints or stop cocks were used in the system. It is believed that the vacuum was as high as is obtainable by such means.

Apparatus and Method

A large spectrometer with two-inch objectives was used as a monochromatic illuminator. The eye-piece with its rack and pinion were removed from the telescope and a special tube substituted into which were fitted a series of four diaphragms with parallel rectangular apertures of decreasing width as the final slit was approached. The inside of this

[3] C. C. Bidwell, Phys. Rev. **23**, 357 (1924)
[4] H. E. Ives, Jour. Opt. Soc. Amer. and Rev. Sci. Inst. **8**, 551 (1924)

tube and both sides of all the diaphragms were covered with a jet black material having a velvet surface. Likewise the prism house was lined with black cotton flannel. No stray light could possibly enter the slit as a result of multiple reflections, the beam of light being narrowly confined to the axis of the telescope. A 60-degree flint glass prism was used, having faces three inches square.

Several sources of radiation were tried. The one finally adopted and used throughout this work was a 600-watt, 20-ampere, gas-filled lamp with a V-shaped, helical filament. The current was maintained at a remarkably constant value by means of an auxiliary storage battery, connected in parallel with the lamp and an adjustable low resistance. A low-range ammeter in series with this auxiliary battery served as a highly-sensitive control for the current through the lamp. The main rheostat, in series with the lamp, was water cooled.

The energy of the light was measured by means of a Coblentz thermopile and a Leeds and Northrup high-sensitivity galvanometer. The thermopile was mounted upon the slit in front of the photo-electric cell so that it could be moved into position for checking the calibration at will. The scale deflections for wave-lengths near the long-wave limit were as large as six or seven centimeters, decreasing to a few millimeters for λ 4359. The energy readings were highly satisfactory. Being entirely free from disturbing effects, even the smallest deflections could be repeated to one per cent. Throughout the work, slit widths of one-hundredth of an inch were used.

The photo-electric currents were measured with a Dolezalek electrometer, having a sensitivity of 2,800 mm per volt. All currents reported are saturation values. The potassium electrode was maintained at a negative potential of 120 volts.

The arrangement for temperature control of the cell is indicated in Fig. 1. The cell was supported by brass brackets within a double-walled copper box, $5\frac{1}{2}$ inches square on the inside and 6 inches square on the outside. By means of a pressure siphon, the liquid air was pumped into the air-tight cavity between the inner and outer walls of the box. It was introduced through a glass tube at the top of one end and the vapor escaped through a small glass tube at the top of the opposite end. The copper box containing the cell was mounted in the center of a wooden box twelve inches square, so that a layer of pure wool three inches thick and considerably compressed, surrounded the copper container on all sides. The beam of light, which came to a focus at the slit and thermopile, diverged through a distance of about six inches and illuminated a good-sized area on the potassium surface.

In order to minimize heat conduction, the beam of light was led to the cell through a glass tube, three inches long and one inch in diameter, on each end of which was sealed a thin plate glass window. The windows were sealed on with a pasty mixture of water glass and talcum, a cement which Oldenberg[5] found would hold a vacuum even when the joint was immersed in liquid air. This tube has held a good vacuum for a period of three months during which it has been cooled repeatedly to the tempera-

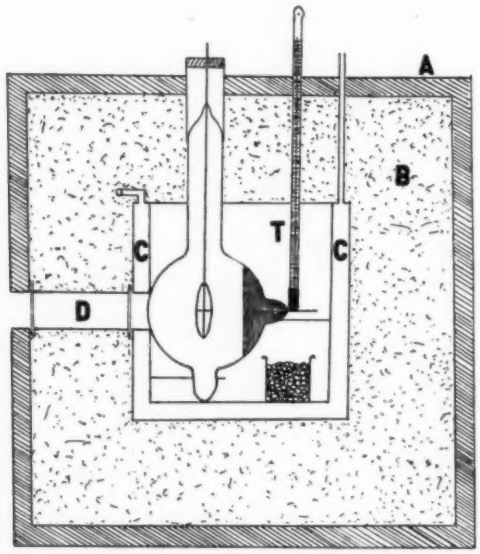

Fig. 1. The arrangement for temperature control of the cell. *A*, outer wooden box; *B*, wool packing; *C*, cavity for liquid air; *D*, evacuated tube; *T*, pentane thermometer.

ture of liquid air. Frost formation on the inner window was prevented by a small beaker of drying material. The outside window was kept clear of frost by blowing dry air upon it. Temperatures were measured with a pentane-in-glass thermometer whose bulb rested in contact with the heavy tungsten wire leading through the cell wall into the block of potassium. The heat insulation of the apparatus is indicated by the fact that two liters of liquid air would cool the cell down to −180°C and hold it for over two hours.

Results and Conclusions

The curves in Fig. 2 are typical. They show the shift in the photoelectric currents per unit light intensity between room temperature and the temperature of liquid air. The perfect reversibility of the effect is

[5] O. Oldenberg, Ann. der Phys. **67**, 258 (1922)

indicated by the fact that the points for two separate runs, several days apart, are plotted on each curve. These curves could be repeated at will. Fig. 3 shows these same curves in the region of the long-wave limit with the ordinates magnified twenty-seven times. The long-wave limit is not well defined, but there is no doubt that it shifts along with the curves. During the progress of the work the cell was cooled down with liquid

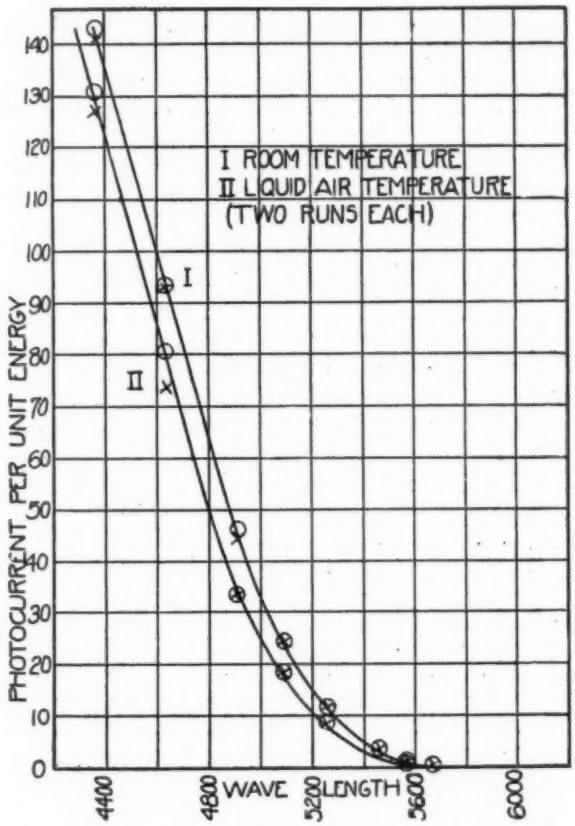

Fig. 2. Photo-current curves (I) at room temperature and (II) at liquid air temperature.

air fourteen times. A study of the tables of data for the best runs at each temperature led to the decision that the long-wave limit was about 6500 A at 20°C and about 6000 A at −180°C. But these values are subject to an error in either direction of 100 A, or more, because of the slow approach of the curves to the wave-length axis. The corresponding values reported by Ives are 7000 A and 6200 A. That the asymptotic character of the curves in the region of the long-wave limit can be due to

stray radiation is doubtful in view of the precautions taken in these experiments. It seems to be characteristic of pure potassium in a high vacuum. At any rate the recent work of Ives and earlier observations by Pohl and Pringsheim point also to this conclusion.

The curves in Fig. 2 do not indicate a selective effect. As a matter of fact, preliminary observations with shorter wave-lengths gave evidence of a maximum value at about 4200 A. In this region, however, the absorption of the glass prism was so great that the thermopile gave a galvanometer deflection of less than 2 millimeters, making the data too uncertain to warrant conclusions with regard to the temperature effect.

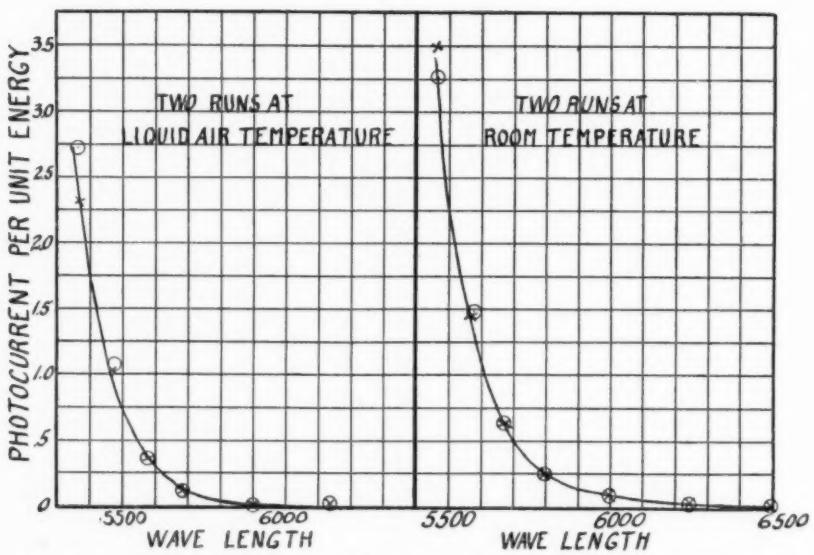

Fig. 3. Curves showing long wave limit at room temperature and at liquid air temperature.

The arrangement for temperature control of the cell was designed with the idea of holding the temperature indefinitely at any point, so that the photo-electric response could be traced in detail from −180°C to 20°C. Indeed, the temperature could be easily held constant to one-half a degree by pumping in a small quantity of liquid air whenever the thermometer indicated a slight creeping up of the temperature. After many attempts, however, this method of taking readings had to be given up because the results were inconsistent. The *sudden chilling* of the metal affected the photo-electric currents and, in general, gave abnormally high values. When liquid air was introduced with the cell at room temperature, the photo-currents for all wave-lengths always increased decidedly, and the

response was sudden. This increase, as Ives has pointed out, was undoubtedly due to the condensation of potassium vapor and the formation of a partially new surface. However, the rate of fatigue of this newly deposited metal was not constant for different trials. It apparently depends upon the amount of liquid air which is pumped in at the start. Furthermore, the process of holding the cell at a given temperature, namely, by introducing a small quantity of liquid air at frequent intervals, raised the photo-currents above their normal values. This effect is not understood. It is possibly due to the fact that the mirror surface of the potassium was broken by check lines into a large number of small polygons. The sudden chilling of the metal and the consequent

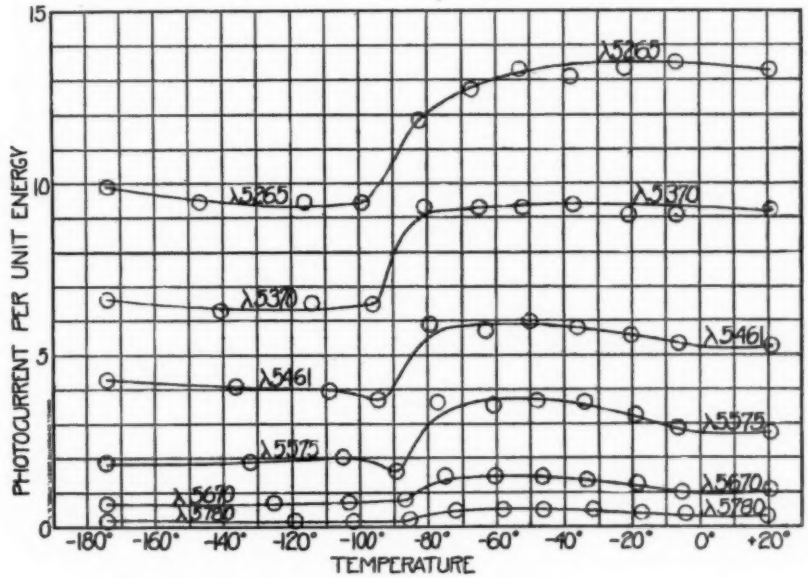

Fig. 4. Photo-current as function of temperature, for various wave-lengths.

contraction would conceivably open up these cleavage lines of the surface and allow the incident light to reach down more deeply into the surface layer. It should, however, be pointed out that when the cavity of the box was filled with liquid air and kept full until equilibrium was obtained, the resulting photo-currents at $-180°C$ were constant and repeatable.

A certain procedure gave fairly consistent results and apparently revealed one of the contributing causes of the shifts of curves like those shown in Figs. 2 and 3. This procedure was merely to cool the cell to equilibrium at $-180°C$, then take readings of photo-currents and corresponding temperatures while the apparatus gradually warmed up to room

temperature. A set of curves so obtained is shown in Fig. 4. Temperatures were read immediately before and after the photo-electric currents, and mean values plotted in the curves. The temperature rise during the time necessary to read the electrometer deflection and check it amounted to two or three degrees at the start and gradually decreased to about one degree at $-80°C$ and to about one-half degree at $+20°C$.

An abrupt transformation point is evident in all of the curves. On each side of this point the curves are nearly parallel to the temperature axis. For shorter wave-lengths a similar though less prominent discontinuity was observed. It was not merely an accidental irregularity for it occurred repeatedly and persisted even after removing the cell from the box and melting down the potassium to form a new surface. As a change in crystal structure is known to occur in this region, correlation of the two effects is at once suggested, especially since Bidwell found a sudden change in the thermo-electric power at approximately the same temperature. However, this method of locating the discontinuity, if it is a real effect, is open to the objection that there is a temperature gradient of an unknown amount between the illuminated surface of the metal and the tungsten electrode which rested in contact with the bulb of the thermometer; moreover the cell as a whole is not in temperature equilibrium while the liquid air is boiling out of the container. The possible connection with crystal structure, therefore, requires further study, not only with potassium but also with other alkali metals.

This investigation, then, while it points definitely to the reality of the temperature effect, suggests the interesting probability that the shift of the long-wave limit is due, in part at least, to the change in crystal structure.

This experimental work was carried out at Cornell University during the winter of 1923-24, on sabbatical leave of absence from Carleton College. I am deeply indebted to Professor Merritt and other members of the Department staff for many courtesies. In particular, I wish to thank Professor F. G. Tucker, whose timely suggestions and constant interest were especially helpful.

CARLETON COLLEGE,
June 23, 1924.

THE THEORY OF THE DECAY OF ALPHA RAY LUMINESCENCE

By Enos E. Witmer*

Abstract

The Walsh equation for the decay of luminosity in substances which luminesce under α-ray bombardment, $I = I_\infty + I_0 e^{-\lambda t}$ is modified by introducing the additional assumption that the field of radiation due to the decay of the active centers reacts on the centers in the inactive state and forms new active centers by reversing the decay process. The new equation $(I - I_\infty)/(I + I_1) = C e^{-\lambda t}$ fits data obtained by Miss F. Rodman (Phys. Rev. **23**, 478, 1924) for the decay of luminosity of c.p. radium bromide, which the Walsh equation does not. It is only approximately correct when the radiation effect is small. The exact equation developed on these assumptions, which is more complicated, is also given. For a *mixture of radium bromide and barium bromide* the data are well represented by the sum of two time-luminosity curves with equations of the less exact type.

PERRIN'S well known hypothesis[1] that the initiation and course of molecular reactions is essentially connected with the flood of radiation in which the reacting system is immersed has an obvious bearing on the phenomena of luminescence generally. Perrin himself assumed that substances which exhibit these phenomena possess two or more stable molecular states, and that radiation of definite frequency or frequencies causes these systems to pass from one state to the other with the simultaneous emission of radiation of another frequency. This idea is concisely given by the formula

$$A + h\nu \rightleftarrows B + h\nu',$$

where A and B represent the initial and final configurations of a reaction initiated by the frequency ν and accompanied by the emission of the frequency ν'.

Perrin discusses the application of this notion to phosphorescence, fluorescence, and thermoluminescence. It is evident that a similar application can be made to the luminescence exhibited by some substances under α-ray bombardment. The intensity of the luminosity in this case decreases with the time of exposure to the α-rays. Rutherford explained this decay by assuming the presence in the substance of active centers which were destroyed by the α-ray bombardment with the emission of light. The luminosity was consequently proportional to the

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[1] Perrin, Ann. de Physique **11**, 5 (1919)

rate of decay of the active centers, that is to the number of those centers present. With constant α-radiation this gives an exponential time-luminosity decay curve. Walsh[2] found, however, that in certain substances the luminosity did not decay to zero but approached a finite limiting value. To explain this Walsh added to Rutherford's theory the "recovery hypothesis," i.e. that the active centers were being continually re-formed at a rate proportional to the number of centers in the inactive state. This gave him as the equation for the decay of luminosity I,

$$I = I_\infty + I_0 e^{-\lambda t}, \tag{1}$$

which agrees with his results.

These theories offer no suggestion as to the nature of an active center. It seems natural to suppose, however, that such a center is a molecular or atomic system in a higher quantum state, which, although stable, is not the most stable configuration of the system under the existing conditions. This would explain the rejuvenation of the luminosity of certain of these substances, such as radium bromide, on the application of heat. The heating throws the molecular systems into higher quantum states and thus re-forms the active centers.

With this conception of an active center in mind, a direct application of the Perrin hypothesis can be made. The radiation produced by the decay of the active centers might react on the inactive centers with the consequent formation of new active centers. The field of radiation produced by the substance would thus influence the rate of formation of the active centers. It is conceivable, of course, that, contrary to the Perrin hypothesis, the radiation might react on the active centers and influence their rate of decay, or that the radiation might influence both the rate of decay and the rate of formation of the active centers.

This paper is concerned with the mathematical development of the idea outlined above. Let us assume first, in accordance with Perrin's hypothesis, that the radiation affects only the recovery process.

Let $n \equiv$ the number of active centers at the time t;
$n_0 \equiv$ the initial number of active centers;
$n_\infty \equiv$ the final number of active centers;
$N \equiv$ the total number of centers whether in the active or inactive state;
$I \equiv$ the luminosity at the time t;
$I_\infty \equiv$ the minimum limiting luminosity;
$\beta n \equiv$ the number of active centers which decay in unit time;
$\alpha(N-n) \equiv$ the number of active centers which form in unit time according to the Walsh hypothesis, α and β being constants.

[2] Walsh, Proc. Roy. Soc. (A) **93**, 550 (1917)

Now assume that $pI(N-n)$ active centers are formed in unit time due to the influence of the radiation in which the system is immersed. If the number of active centers formed by this reverse process is a small fraction of the active centers which decay, we may neglect the radiation absorbed in this way and take $I=k\beta n$, where k is a constant. Hence $\gamma n(N-n)$ = the number of active centers formed per unit time due to radiation, γ being a constant.

Then
$$dn/dt=(\alpha+\gamma n)(N-n)-\beta n \tag{2}$$
$$=-\gamma n^2+(\gamma N-\alpha-\beta)n+\alpha N=-\gamma(n-n_\infty)(n+a), \tag{3}$$

where
$$n_\infty, -a=[\gamma N-\alpha-\beta\pm\sqrt{(\gamma N-\alpha-\beta)^2+4\alpha\gamma N}\,]/2\gamma. \tag{4}$$

Since α, β, and N are necessarily positive, we see that if γ is positive, n_∞ and a are positive, but if γ is negative, n_∞ is positive and a negative.

The solution of this equation is

$$\frac{n-n_\infty}{n+a}=\frac{n_0-n_\infty}{n_0+a}e^{-\gamma(n_\infty+a)t}, \tag{5}$$

whence
$$\frac{I-I_\infty}{I+I_1}=\frac{I_0-I_\infty}{I_0+I_1}e^{-\gamma(n_\infty+a)t}\equiv Ce^{-\gamma(n_\infty+a)t}, \tag{6}$$

or
$$\log\frac{I-I_\infty}{I+I_1}=-\gamma(n_\infty+a)t+\log C \tag{7}$$

I_1 being a constant.

That this is a generalization of the Walsh equation may be shown by writing it in the form

$$I=I_\infty+(I_\infty+I_1)\frac{Ce^{-\gamma(n_\infty+a)t}}{1-Ce^{-\gamma(n_\infty+a)t}}. \tag{8}$$

If, taking account of the radiation absorbed, we had written

$$I=k\beta n-k'pI(N-n), \tag{9}$$

the resulting equation for n would have been of the form

$$\frac{(n-n_\infty)^{1-\lambda_3}}{(n-a)^{1-\lambda_3}}=Ce^{-\lambda t} \tag{10}$$

Since from (9)
$$n=\mu I/(1+\mu I_1) \tag{11}$$

the resulting equation for I is more complicated than (10).

The assumption that the field of radiation affects both the decay and the recovery processes in the manner indicated above results again in an equation of the type of (10) with the general relation between n and I indicated in (11).

Eq. (7) shows that plotting $\log [(I-I_\infty)/(I+I_1)]$ against time t should give a straight line if I_1 is properly chosen.

It has been stated that the Walsh equation covers the particular data which he collected. One would expect the radiation effect considered in developing formula (7) to be most prominent where the initial luminosity is great. It is interesting, therefore, to observe that the luminosity decay curve of c.p. radium bromide, which has a high initial luminosity, does not conform to Walsh's equation.[3] Fig. 1 shows the straight line obtained

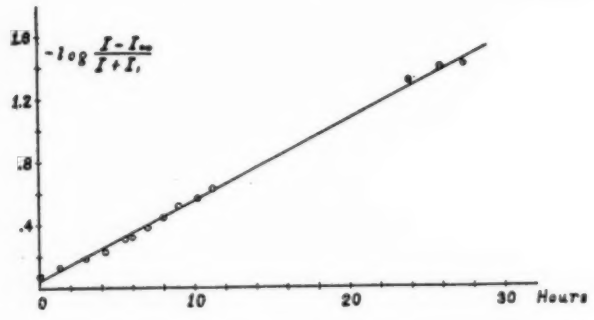

Fig. 1

by plotting according to Eq. (7) data kindly furnished by Dr. Rodman for a tube of c.p. radium bromide (tube no. 1, Miss Rodman's paper). I_1 was taken equal to 1000. The equation of this curve is

$$(I-6)/(I+1000) = .9205\ e^{-.119t}. \tag{12}$$

Hence $I_\infty = k\beta n_\infty = 6$; $I_1 = k\beta a = 1000$; whence from Eqs. (7) and (12) $\gamma = .000118\ k\beta$, and $dn/dt = -.000118\ k\beta n^2 - .117\ n + .708/k\beta$. Comparing this with Eq. (3) we see that the coefficients have signs in accord with the meanings assigned the symbols.

To represent the same data, Miss Rodman used the purely empirical formula

$$I \equiv I_\infty + I_0 e^{-At^B}. \tag{13}$$

In this particular case $B = .67$. This equation is highly unusual in form and difficult to interpret physically. Furthermore it does not give the correct value of the initial luminosity, but as stated by Miss Rodman represents the data only after the first hour; also the first derivative at $t=0$ is infinite in this particular case, which indicates an infinite rate decay at the start.

[3] Rodman, Phys. Rev. **23**, 478 (1924)

Miss Rodman's paper also contains curves for the decay of luminosity of mixtures of radium bromide and barium bromide. Eq. (13) does not represent these curves, neither does Eq. (7) apparently. It seems probable, however, that the curve for a mixture would be a composite curve, i.e. that the luminosity of each component substance would decay at a rate independent of the other to a first approximation. Such a curve could be split into two curves each represented by Eq. (7). The writer has succeeded in doing this for the one set of data for a mixture on which it was tried.

In conclusion it may be stated that the meager experimental evidence available is not inconsistent with the idea that the rate of decay of luminescence in compounds of the class here treated is influenced by the self-developed radiation, more or less in accord with the Perrin hypothesis.

RESEARCH SECTION
RANDAL MORGAN LABORATORY OF PHYSICS,
UNIVERSITY OF PENNSYLVANIA,
July 15, 1924

THE CONNECTIONS BETWEEN THE FOUR TRANSVERSE GALVANOMAGNETIC AND THERMOMAGNETIC PHENOMENA

By P. W. Bridgman

Abstract

Three relations are deduced by reasoning of a general character between the four transverse effects, so that each of the effects may be expressed in terms of any other effect. The first relation, $Q=KP/T$, is obtained from energy considerations by assuming that the source of energy involved in the *Nernst effect* Q is provided by the *Ettingshausen effect* P, K being the thermal conductivity and T the absolute temperature. It is shown that the *Hall effect* R and the *Righi-Leduc effect* S each provides its own source of energy. The second relation, $Q=\sigma R/\rho$, where σ is the *Thomson coefficient* and ρ the specific resistance, is that of Moreau, and is obtained by assuming that the level surfaces of the Thomson e.m.f. in a metal carrying a thermal current are rotated by the Hall effect. The third relation, $P=S\sigma T/K$, involves the concept, probably new, of a temperature gradient generated by a heat current flowing down a difference of electrical potential. This new effect, which is the thermal analog of the Thomson effect, should be capable of experimental detection. It is proposed to call it the *Thomson temperature gradient*. The third relation is obtained by assuming that the isothermal lines associated with the Thomson temperature gradient are rotated by the Righi-Leduc effect. The experimental data available are not very satisfactory but they agree with the above relations within the experimental error.

IN this note, I deduce by reasoning of a general character three relations connecting the four transverse effects. Of these three relations, one has already been proposed by Moreau; the other two have not been published as far as I can find, but one of them was independently deduced by Professor H. A. Lorentz at practically the same time as by me, both of us being stimulated by the approaching Solvay Conference on Conduction in Metals.

Although the reasoning by which the relations are deduced is of a general character, it is not completely general, but will impose certain restrictions on any mechanism proposed to give a detailed account of the effects. The general state of affairs is somewhat like that with regard to the application of thermodynamics to the thermo-electric circuit by Kelvin; thermodynamics strongly suggested but did not demand the relations found. The proposed relations will, therefore, need experimental verification. The experimental errors in measuring these effects are known to be very large, but I believe that within experimental error the proposed relations are satisfied.

It will be well first to review briefly the definitions of these effects.[1] All of them are transverse, that is, they take place in a plane perpendicular to a uniform magnetic field, and depend on the field. For simplicity we suppose the plane in the shape of a rectangle.

(1) *The Hall effect.* If an electric current passes lengthwise of the rectangle, then a transverse difference of electrical potential is generated, so that the current no longer flows perpendicular to the equipotential surfaces. The Hall coefficient R is defined by the equation

$$R = Ed / IH ,$$

where E is the transverse potential difference, d the thickness of the plate parallel to H, I the total current, and H the magnetic field intensity. The coefficient is taken as positive if the electrical potential is raised on that side of the plate on which the Amperean current generating the magnetic field has the same direction as the current I.

(2) *The Ettingshausen effect.* When an electric current flows lengthwise of the rectangle, a transverse temperature difference is generated in the magnetic field. The Ettingshausen coefficient P is defined by the equation

$$P = \Delta T \times d / IH ,$$

where ΔT is the transverse temperature difference. It is positive by convention if the plate becomes warmer on that side where I and the Amperean current have the same direction.

(3) *The Nernst effect.* If a heat current flows lengthwise of the rectangle, a transverse electrical potential difference is generated in the magnetic field. The Nernst coefficient Q is defined by the equation

$$Q = EKd / WH ,$$

where K is the thermal conductivity, and W the total amount of heat flowing through the cross section of the plate per second. By convention Q is positive if the electrical potential is raised on that side of the plate where the Amperean current and the heat current have the same direction.

(4) *The Righi-Leduc effect.* If a heat current flows lengthwise of the plate, a transverse difference of temperature is established in the magnetic field. The Righi-Leduc coefficient S is defined by the equation

$$S = \Delta T \times Kd / WH$$

[1] The notation, definitions, and the source of the numerical data used in the following, when not explicitly given otherwise, will all be found in the recent very comprehensive book on this subject by L. L. Campbell, entitled Galvanomagnetic and Thermomagnetic Effects, published by Longmans, Green and Co., 1923.

S is positive by convention if the plate is warmer on that side on which the Amperean current and the heat current are in the same direction.

These four effects are thus seen to involve either a difference of electrical or thermal potential. Such differences may serve as sources of energy, by allowing either an electrical current to flow between the points having a difference of electrical potential or a heat current between the points having a difference of temperature. One connection which we are seeking between the effects may be found by inquiring what is the source of energy of these possible transverse currents.

We consider first the Hall effect, because it indicates most simply the character of the relations to be assumed, although it does not immediately yield any new information. In a metal, which is the source of a Hall e.m.f. under a current I, allow a transverse current i to flow. The transverse e.m.f. is RIH/d, so that the work per unit time extracted by i is $RIHi/d$. The source of this energy is to be found in the longitudinal Hall e.m.f. set up by the transverse current i. The amount of this e.m.f. is RiH/d, and the current which flows through this e.m.f. is I, making a total additional energy development by I per unit time equal to $RiHI/d$, the amount in question. An examination of the signs shows that they are as they should be. Thus we see that the Hall effect supplies its own source of energy.

Next consider the source of energy in the Nernst effect. The longitudinal heat current W generates a transverse electrical potential difference QWH/Kd, and if an electric current i flows across the plate, the energy it receives per unit time is $QWHi/Kd$. Following the hint given by the Hall effect, we seek for the source of this energy in the Ettingshausen effect of the current i. The transverse current i gives rise to a longitudinal temperature difference PiH/d, and the work which the heat current W does in flowing through this temperature difference is, by the second law of thermodynamics, $W(PiH/d)/T$, where T is the absolute temperature. Equating these two amounts of work we get

$$Q = KP / T , \tag{1}$$

which is the first of the equations of connection desired. An examination of the signs shows that they are as they should be. This was the equation also found by Professor Lorentz.

If this relation should prove to be correct it would mean that any proposed mechanism must be such that a transverse electric current, in virtue of the longitudinal difference of temperature created by it, may extract energy reversibly and in the thermodynamic amount from the

primary heat current (*not* from the miscellaneous heat energy of the surroundings).

The source of energy involved in the Ettingshausen effect is, of course, by the inverse of the above, the Nernst effect, so that there is no new information here. Further, the Righi-Leduc effect provides its own source of energy, like the Hall effect, and similarly yields no information.

Of the two remaining relations, we give first that of Moreau. The guiding idea is that the essence of the Hall effect consists in a definite angular rotation of the electrical equi-potential lines in the magnetic field, irrespective of the origin of the equi-potential lines.

The angle of rotation of the lines may easily be shown to be RH/ρ, where ρ is the specific electrical resistance of the metal.

Consider the plate of the Nernst effect, carrying a longitudinal heat current. The longitudinal temperature gradient θ driving this heat flow is

$$\theta = W / Kbd ,$$

where b is the breadth of the plate. Now this longitudinal temperature gradient is associated with a longitudinal electrical gradient determined by the ordinary Thomson coefficient σ. This electrical potential gradient is equal to $\sigma\theta$. The connection is obtained as follows. If the current I flows between two points unit distance apart in a metal in which there is a temperature gradient θ, then the extra work of the current is $\sigma\theta I$, by definition of σ; hence the effective potential difference is $\sigma\theta$. The longitudinal electrical potential gradient in the Nernst plate is therefore $\sigma\theta = W\sigma/Kbd$. Now the equipotential lines associated with this gradient are rotated through the angle RH/ρ by the Hall effect, giving rise to a transverse electrical potential difference $(W\sigma/Kbd)\ (RH/\rho)b$. But the transverse electrical potential difference generated in this way is nothing but the Nernst potential difference for which we have the expression QWH/Kd. Equating these two, we have

$$Q = \sigma R / \rho , \tag{2}$$

which is the relation of Moreau.

The third relation is to be obtained by considerations analogous to the above but involves (1) the introduction of a concept which is, so far as I know, unfamiliar, namely that of a longitudinal temperature gradient which is generated by a flow of heat down an electric potential gradient, and (2) a somewhat modified description of the Thomson effect.

Consider a metal in which there is a flow of heat under a thermal gradient and in the same direction a flow of electricity under an electrical potential gradient. There are involved here two irreversible processes,

namely flow of heat against the thermal resistance of the metal, and flow of electricity against the electrical resistance; these two processes demand temperature and electrical gradients proportional to the temperature and electrical resistances. But in addition there is a reversible transfer of energy between the thermal and electrical currents. Now the only thing which can feed energy to an electrical current is an e.m.f. or difference of potential, and the only way by which a heat current can deliver work is by flowing through a difference of temperature. Hence superposed on the thermal and electrical potential differences which drive the current against resistance, there must be additional temperature and electrical potential differences corresponding to the reversible transfer. The electrical potential difference is well known, and is merely the ordinary Thomson e.m.f. The corresponding additional temperature gradient seems not to be usually considered. I propose to call it the *Thomson temperature gradient*, and denote it by the letter σ'. The definitions of the two coefficients are entirely analogous.

Electrical energy per unit time in unit length $= I\Delta E' = I\sigma\Delta T$.

Thermal energy per unit time in unit length $= W\,\Delta T'/T = W\sigma'\Delta E$.

Here ΔE and ΔT are the principal electrical potential and temperature differences that drive the currents against resistance, and $\Delta E'$ and $\Delta T'$ are the small superposed differences involved in the reversible effects. These two energies are to be put equal. Now $W = K\Delta T bd$, and $I = \Delta E bd/\rho$ Substituting, we get

$$\sigma' = \sigma/K\rho,$$

and

$$\Delta T' = (\sigma T/K\rho) \times \text{elec. potential diff.}$$

We digress for a moment to remark that the difference between this and the ordinary statement of the Thomson effect is in the more detailed account of the source of the energy absorbed by the electrical current in flowing through a temperature gradient. The source is usually supposed to be merely the miscellaneous heat energy of the surroundings; I have supposed it to be the primary heat current delivering energy reversibly (as in any thermodynamic engine) through a small superposed temperature gradient. This Thomson temperature gradient should be capable of experimental detection, and I propose to search for it. There are interesting questions here: Is there any surface temperature phenomenon corresponding to the supposed electrical double layers? Is there for temperature phenomena anything corresponding to the distinction between an electrical potential difference and an impressed e.m.f.? In this paper I have used these last two terms loosely, but I think without obscuring the meaning.

Returning now to the main argument, we postulate that the essence of the Righi-Leduc effect is a rotation by the magnetic field of the isothermal lines by a definite amount irrespective of their origin. This rotation may be easily shown to be SH. Consider now an electric current flowing longitudinally in a plate. In virtue of the electrical potential gradient $I\rho/bd$ which drives the current, there is set up a longitudinal Thomson temperature gradient of amount $(\sigma T/K\rho)\ (I\rho/bd)$. The corresponding isothermal lines are rotated through the angle SH, giving a transverse temperature difference $(\sigma T/K\rho)\ (I\rho/bd)\cdot SH\cdot b$. But this transverse temperature difference is nothing but the Ettingshausen temperature difference, for which we have the expression PIH/d. Equating these gives

$$P=S\sigma T/K \tag{3}$$

which is the last of the relations required.

These relations may be written in various forms. Thus we may express all the coefficients in terms of the Hall coefficients

$$P=(T\sigma/K\rho)R \tag{4}$$

$$Q=(\sigma/\rho)R \tag{5}$$

$$S=(1/\rho)R. \tag{6}$$

Or we may get an important relation between all four effects not involving σ:

$$PR/QS=\rho T/K. \tag{7}$$

Equation (6) above is merely the statement that the angle of rotation of the isothermal lines is the same as that of the electrical equipotentials. This relation has been previously suggested (Campbell, page 238) but I believe on purely empirical grounds, with no theoretical basis.

When it comes to the experimental verification, we are very much embarrassed by the absence of sufficient data. The effects are very difficult to measure, not only because of their smallness, but because the great numbers of secondary effects are difficult to eliminate. Observers seldom agree and may differ even as to the sign. The effects depend greatly on the state of purity and mechanical working of the specimen. All of the quantities involved in the relations should be measured on the same specimens, and even then we have no security, because the secondary effects may not have been eliminated or the metal may be crystalline in character with different properties in different directions.

There is a very complete collection of numerical results in Campbell's book. From this I have selected the data which are adapted to compari-

son, consisting mostly of measurements by Unwin of the four effects on the same samples.

The results are shown in Table I, in which are given, at a temperature roughly 300° K, the four coefficients R, P, Q, and S, together with K and ρ. The electrical quantities are in absolute e.m.u. From these data KP/T is calculated, which by Eq. (1) should be the same as Q, and the combinations PR/QS and $\rho T/K$, which should be equal to each other.

TABLE I

Metal	R	P	Q	S	K
	($\times 10^{-4}$)	($\times 10^{-9}$)	($\times 10^{-4}$)	($\times 10^{-7}$)	($\times 10^{7}$)
Ag.....	−8.4	−1.65	−1.8	−2.7	4.2
Al.....	−4.0	+1.06	+.42	−0.62	2.1
Cd.....	+8.8	−2.9	−1.2	+0.89	.93
Co.....	+24.5	+21.6	+7.8	+1.1	.60
Cu.....	−5.5	−1.6	−1.9	−2.1	3.8
Fe.....	+87.	−42.6	−9.5	+5.2	.60
Ni.....	−29.	+30.3	+10.	−2.5	.60
Zn.....	+7.6	−2.67	−.73	+1.1	1.11
Au*.....	−6.5	−.96	−1.7	−2.5	2.9
Sb†.....	+2190.	+1940.	+176.	+20.1	.167
Bi†.....	−63300.	+35000.	+1780.	−20.5	.081

Metal	ρ	KP/T	$\rho T/K$	PR/QS
	($\times 10^{3}$)	($\times 10^{-4}$)	($\times 10^{-3}$)	($\times 10^{-3}$)
Ag.....	1.66	−2.2	12	28
Al.....	2.9	+.74	41	163
Cd.....	7.5	−.90	240	240
Co.....	9.7	+4.3	480	620
Cu.....	1.8	−2.0	14	22
Fe.....	12.0	−8.5	600	750
Ni.....	11.8	+6.1	590	350
Zn.....	6.1	−.99	165	250
Au*.....	2.42	−.93	25	15
Sb†.....	40.5	+110.	7,250	12,000
Bi†.....	160.	+950.	59,400	500,000

* From unpublished data of Professor E. H. Hall. See the forthcoming Solvay Conference Report.

† From data by Zahn, given in Campbell's book.[1]

The agreement must, I believe, be considered within experimental error in all cases, except possibly that of bismuth, which we are probably justified in neglecting because of the strongly crystalline character of the metal and the fact that the coefficients depend in a great degree on the strength of the magnetic field.

The two relations checked in the Table have been so chosen that the Thomson coefficient σ does not appear. In checking any third relation, into which σ must enter, we are badly off, because σ varies from specimen to specimen and was not determined for the specimens above.

The best data for this are probably to be found in Moreau's original papers (page 228 of Campbell). The following table is reproduced from Campbell, using Moreau's redetermined Q for cobalt.

TABLE II

Metal	Q(obs.)	Q(calc.)
Bi	+0.196	+0.149
Sb	+ .0094	+ .0090
Ni	+ .0073	+ .0026
Co	− .00146	− .00175
Fe	− .00156	− .00156
Steel	− .00060	− .00062
Cu	− .000073	− .000084
Zn	− .000054	− .000046

The agreement is certainly all that could be asked. The extreme variability of the coefficients in different samples may be seen by comparing Moreau's values for Q with those given in the table above.

Of the metals in Table I it is probable that Ag, Au, and Cu are those on which different observers might be expected to agree most closely. I have determined σ for these metals.[2] If we use my values for σ and calculate $\sigma R/\rho$ which is equal to Q by Moreau's formula, we shall find -1.3×10^{-4}, $-.75\times10^{-4}$, and $-.90\times10^{-4}$ against the respectively observed values of Q -1.8×10^{-4}, -1.7×10^{-4}, and -1.9×10^{-4}. The agreement is probably all that can be expected.

In view of the extreme uncertainty of the experimental evidence and the plausibility of the theoretical argument, I believe that we must consider the theoretical relations as verified, and in fact we can probably now calculate the more difficult of these effects from the easier ones with a result probably more correct than can be obtained by direct experiment.

THE JEFFERSON PHYSICAL LABORATORY,
HARVARD UNIVERSITY,
August 22, 1924.

[2] P. W. Bridgman, Proc. Amer. Acad. **53**, 269-386, (1918)

THE THEORY OF GALVANOMAGNETIC PHENOMENA IN FLAMES

By C. W. Heaps

Abstract

When positive and negative ions are assumed to move through flame gases under the influence of mutually perpendicular electric and magnetic fields and a retarding force of the nature of viscous resistance, the expression derived for magneto-resistance is found not to agree with experiment. It is then assumed that the Hall e.m.f. produced by the deflection of negative ions exerts a dragging effect on the positive ions, which in turn exert a drag on the upward moving flame molecules and thus alter the upward velocity of the flame gases. Allowing for this change of flame velocity, *equations for the Hall effect and for the magneto-resistance effect* are obtained which are in accord with experiment. The Hall effect is shown to be proportional to the difference of mobilities of positive and negative ions (k_1-k_2) divided by $(1+dR/R)$, where dR/R is the magneto-resistance effect.

THE theory of galvanomagnetic phenomena in flames has been worked out by H. A. Wilson[1] and by E. Marx[2]. The expression given by Wilson for the Hall effect is $Y=-k_2H(X-HV)$, where Y is the vertical potential gradient produced in a flame by a magnetic field H normal to Y and to the horizontal potential gradient X. Here k_2 is the mobility of the negative ion and V is the upward velocity of the flame gases. Values of k_2 determined by using this equation range around 20 m per sec. for one volt per cm. Other methods of obtaining k_2 give considerably higher values, ranging from 60 to 200 m per sec. The expression deduced by Wilson[3] for the magneto-resistance of a flame is $dR/R=H^2k_2^2/12-HV/X$. Here dR is the change produced in R, the resistance of the flame, by the magnetic field H acting normally to X and V. This equation, to agree with experiment, requires a value of V about 100 times as large as the value actually observed. It appears that Wilson's theory is inadequate. Marx has criticized Wilson's methods and derived a different equation, but his theory is no more satisfactory in its results than Wilson's. Marx also, in his deductions, neglects the effect of the Hall electromotive force on the magneto-resistance, a neglect which vitally affects his conclusions.

The theory outlined below is somewhat more general than the theories of Wilson and Marx. Also, an expression for the magneto-resistance of a flame is derived which does not conflict with experiment.

[1] H. A. Wilson, Electrical Properties of Flames, p. 113
[2] E. Marx, Handbuch der Radiologie, p. 749
[3] H. A. Wilson, opus cit.[1], p. 117

Let axes be chosen in a long flat flame so that H is along the z axis perpendicular to the flame, X is along the horizontal x axis in the plane of the flame, and Y and V are along the vertical y axis. Let u_1 and v_1 be the drift velocities of the positive ions along the x and y axes, respectively, under the influence of the electric fields. The corresponding drift velocities of the negative ions are represented by u_2 and v_2. Also let k_1 be the mobility of the positive ions. In the steady state we may set the sum of all forces on an ion in any one direction equal to zero. Hence

$$Xe+Hev_1-eu_1/k_1=0\ ; \tag{1}$$

$$Ye-Heu_1-e(v_1-V)/k_1=0\ ; \tag{2}$$

$$-Ye+Heu_2-e(v_2-V)/k_2=0\ ; \tag{3}$$

$$-Xe-Hev_2-eu_2/k_2=0\ . \tag{4}$$

These equations imply a viscous resistance to the motion of ions through the flame gases. Solving we obtain

$$u_1=\frac{k_1(X+HV+k_1HY)}{1+H^2k_1^2}\ ; \tag{5}$$

$$v_1=\frac{V+k_1(Y-k_1HX)}{1+H^2k_1^2}\ ; \tag{6}$$

$$u_2=\frac{-k_2(X+HV-\ k_2HY)}{1+H^2k_2^2}\ ; \tag{7}$$

$$v_2=\frac{V-k_2(Y+k_2HX)}{1+H^2k_2^2}\ . \tag{8}$$

These equations are the same as those given by Townsend's theory[4] if we let $k_1=T_1e/m_1$ and $k_2=T_2e/m_2$, the subscripts 1 and 2 referring to the positive and negative ions, respectively, and T representing the free period of an ion.

If the top and bottom of the flame are insulated so that no vertical current can flow we have $n_1v_1=n_2v_2$, where n_1 and n_2 are the respective densities of positive and negative ions. If there is no free charge in the flame $n_1=n_2$, so that $v_1=v_2$. Equating the two expressions for v_1 and v_2 above and solving we get

$$\frac{Y}{HX}=(k_1-k_2)\frac{1+VH/X}{1+H^2k_1k_2}\ . \tag{9}$$

[4] Townsend, Electricity in Gases, p. 100

This is the expression for the Hall effect. Usually the terms VH/X and $H^2k_1k_2$ will be very small.

The expression for the resistance change due to the magnetic field is now easily obtained. Substituting in Eq. (7) the value of Y given by (9), we obtain

$$u_2 = \frac{-k_2(X+HV)}{1+H^2k_1k_2}. \tag{10}$$

The mobility of the positive ion in a flame is much less than that of the negative ion so it is usual to assume that practically all the current is carried by negative ions. Hence if I is the current density and R the specific resistance of the flame, we have $I=X/R=n_2eu_2$. Thus $1/R=n_2ek_2(1+HV/X)/(1+H^2k_1k_2)$.

If R', n_2', k_2' indicate values of the respective quantities when $H=0$, we have $1/R'=n'_2ek_2'$, and

$$\frac{dR}{R'} = \frac{R-R'}{R'} = \frac{n_2'k_2'}{n_2k_2}\left(\frac{1+H^2k_1k_2}{1+HV/X}\right)-1. \tag{11}$$

If a magnetic field does not change the rate of recombination or ionization we may write $n_2=n_2'$. We may also assume $k_2=k_2'$ for the following reason. k_2 depends on the mass and the mean free period of the ion. The mean free period is the mean free path divided by the velocity of agitation. A magnetic field would convert large numbers of straight free paths into curved paths, but if the mean free path is obtained in the usual way the result is independent of the curvature of the path. Also the velocity of agitation is unchanged by H and hence the free time is unchanged. We would not expect a magnetic field to change the average mass of an ion hence we may assume $k_2=k_2'$. Eq. (11) may then be written in the form

$$\frac{dR}{R'} = \frac{H^2k_1k_2-HV/X}{1+HV/X} \tag{12}$$

The equation fitting the experimental results[3] is $dR/R=3.1\times10^{-9}H^2+1.5\times10^{-5}H$. Neglecting HV/X compared with unity in the denominator of Eq. (12), we may write this equation in the form $dR/R=H^2k_1k_2-HV/X$. Comparing the two equations for dR/R we get $k_1k_2=3.1\times10^{-9}$ and $V/X=1.5\times10^{-5}$. Experiments indicate that to a fair approximation $k_1=10^{-3}k_2$. Hence we get $k_2=176$ m per sec. for one volt per cm. This value is much too large if other experiments are to be relied upon. Also, letting $X=10^9$, the approximate value for which the experimental equation was determined, we find $V=150$ m per sec. This value is about 100 times too large.

The difficulty with the theory appears to lie in the assumption that the upward velocity of the flame gases is constant. If the magnetic field acts on the horizontal current of negative ions with, say, a downward deflecting force, then these negative ions move downward until the Hall e.m.f. is developed. This Hall e.m.f. will urge positive ions in a downward direction and since the positive ions are supposed to be large and numerous it is evident that they will exert a downward drag on the upward moving flame gases. The following forces in the y direction may be supposed to act on the neutral flame molecules in a unit cube: (1) $\partial P/\partial y$, where P is the pressure of the flame gases, (2) $n_1 e(v_1 - V)/k_1$, representing the viscous resistance to motion of the positive ions through the flame gases, (3) the forces of viscosity tending to retard any relative motion of the flame gases.

In the case of (3) if we think of the air outside the flame as retarding its upward motion much as the walls of a solid tube would retard it, the velocity gradient along z in the flame would be proportional to V. The gradient along x may be assumed zero. The force (3) may therefore be represented by $-AV$, where A is some constant. Hence if $B = n_1 e/k_1$ we may write $B(v_1 - V) - AV - \partial P/\partial y = 0$. Let $V = V_1$ when $v_1 = V$. Then $\partial P/\partial y = -AV_1$ and we get $V = Bv_1/(A+B) + AV_1/(A+B)$, or

$$V = Cv_1 + V_0 \tag{13}$$

where $C = B/(A+B)$, and $V_0 = AV_1/(A+B)$. The term containing C is by no means negligible. J. S. Watt has observed[5] that when a current of about 40 milliamperes is sent through a flame like the one considered above (but into which salt is sprayed), then if H is about 5000 gauss the value of V is made almost zero for one direction of H. If either the current or the field is reversed the flame velocity is much increased. In other words, changing the sign of v_1 very appreciably affects V.

If in Eqs. (6) and (8) we substitute the value of V given by (13), equate v_1 and v_2 as before, and solve the resulting equation, we get

$$\frac{Y}{XH} = (k_1 - k_2)\left(\frac{1 - C + V_0 H/X}{1 - C + k_1 k_2 H^2}\right). \tag{14}$$

The terms $V_0 H/X$ and $k_1 k_2 H^2$ are very small, of the order 10^{-3} or 10^{-4} for ordinary magnetic fields. Hence, unless C is nearly unity its presence will produce very little effect on the value of Y/XH. If $1 - C$ is small enough the value of Y/XH should vary slightly with the direction of H. But C will not be comparable with unity unless B is large compared with

[5] J. S. Watt's paper will follow this in the issue for January 1925.

A. A large value of n_1 is associated with a large value of B, so that for strong ionization there should be an asymmetry in the Hall effect. Watt has observed in a strongly salted flame that the Hall effect may be greater for one direction of the field than for the other.

The expression (12) for the magneto-resistance of the flame will now require modification. From Eqs. (3), (4), (13), and (14), remembering that $v_1 = v_2$, we get

$$u_2 = -k_2\left(\frac{(1-C)X+HV_0}{1-C+k_1k_2H^2}\right). \tag{15}$$

Assuming as before that $n_2 = n_2'$ and $k_2 = k_2'$, we get

$$\frac{dR}{R} = \frac{H^2k_1k_2 - HV_0/X}{1-C+HV_0/X}. \tag{16}$$

To compare this equation with experiment we may substitute the following values for the quantities on the right hand side: $k_1 = 2\times10^{-7}$ cm per sec., $k_2 = 10^{-4}$ cm per sec., $X = 10^9$, $V_0 = 100$ cm per sec. The value assumed for V_0 is of course apt to be very much in error. We can only be sure that it is less than 200 cm per sec., the velocity which has been found for freely moving flame gases. The value of X is roughly that used by Wilson in his experiment. The numbers assigned to k_1 and k_2 are what other experiments have indicated to be about correct. If now we arbitrarily take $1-C = 0.006$ and plot Eq. (16) the resulting curve is approximately coincident with the curve drawn by Wilson to represent his experimental results.

It appears, therefore, that C is very nearly unity in the case of a Bunsen flame such as Wilson used. Nevertheless, using the values of k_1 and k_2 given above and taking $H = 5000$, the quantity $1-C$ in Eq. (14) is about 10 times larger than $k_1k_2H^2$. It is also perhaps 10 times larger than V_0H/X so that to a fairly close approximation the expression Y/XH gives the value of $k_1 - k_2$.

The accurate expression for the Hall effect may be written in a more convenient form for experimental work than the form of Eq. (14). From (16) we have

$$1-C = \frac{Rk_1k_2H^2}{dR} - \frac{HV_0R}{XdR} - \frac{HV_0}{X}.$$

Substituting this value of $1-C$ in Eq. (14) and simplifying gives

$$\frac{Y}{XH}\left(1+\frac{dR}{R}\right) = k_1 - k_2. \tag{17}$$

This relationship is independent of C and all the quantities on the left hand side are easily determined experimentally. In calculating $k_1 - k_2$ the value of dR/R must of course be obtained for the same direction of H and X as the value of Y.

The conclusion is, then, that the upward velocity of the flame gases is an important factor in the theory of galvanomagnetic phenomena in flames. This upward velocity may be materially changed by the electrodynamic action of the magnetic field on a current in the flame, but if this effect of the field is allowed for, the resulting equation for magneto-resistance agrees with experiment. The equation for the Hall effect is found to contain a term depending on magneto-resistance.

THE RICE INSTITUTE,
HOUSTON, TEXAS.
August 18, 1924.

MAGNETIC PROPERTIES OF THIN FILMS OF FERROMAGNETIC METALS PRODUCED BY THE EVAPORATION METHOD

By A. J. Sorensen

Abstract

Magnetic properties of thin films of iron, nickel and cobalt.—The films were deposited by the evaporation method on tin or aluminum foil, and the magnetic properties studied by an induction method for fields up to 139 gauss. The thickness of the films varied from 20 to 300 millimicrons (mμ). The results are as follows. (1) The maximum value of the intensity of magnetization is of the same order of magnitude as for bulk metal. In the case of cobalt, however, it is evident from the curve that with higher fields values higher than any heretofore attained might be secured. The magnetization is independent of the crystal size and the thickness of the film. (2) The remanence is high for iron and cobalt. For nickel a low value was obtained, perhaps due to oxidation. The remanence depends on the crystal size but not on the thickness. (3) The coercive force is high for all three metals and changes abruptly to lower values as the thickness is increased beyond a critical value, which for iron, cobalt and nickel is about 55, 70 and 200 mμ respectively. The high values of the coercive force may be due to the minute size of the crystals in the films. (4) The results combined with Steinberg's curves for the Hall effect in films, tend to contradict Kundt's conclusion that the Hall e.m.f. is proportional to the intensity of magnetization.

THE present work is an investigation of the magnetic properties of thin films of iron, nickel and cobalt, similar to those which were investigated in regard to Hall effect by J. C. Steinberg[1] and which in some respects gave remarkable results.

Method and Apparatus

The method used was similar in principle to one used by Kaufmann and Meier,[2] in which the intensity of magnetization was obtained by measuring the change of flux through a search coil, which was situated in a constant magnetic field, when the magnetized specimen was removed from the search coil. The arrangement of the instruments is shown in Fig. 1.

To obtain the sensitiveness required by the very small amount of metal available (one continuous operation of the evaporating device

[1] Steinberg, Phys. Rev. **21**, 22 (1923)
[2] Kaufmann and Meier, Phys. Zeit. **12**, 513 (1911)

yielding only about one milligram of metal), the scheme illustrated in Fig. 2 was adopted.

In Fig. 2, the switches S_1 and S_2 are both in series with the galvanometer and search coil. The rubber band R is under considerable tension

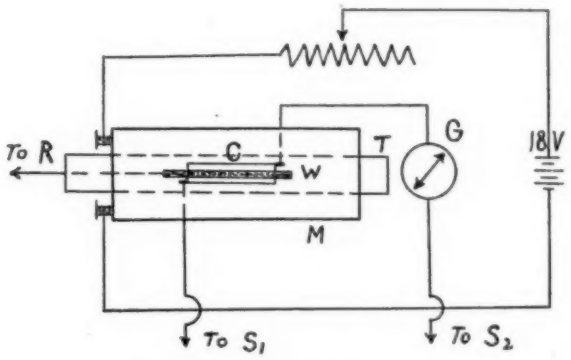

Fig. 1. Experimental arrangement.

and can be held in the position shown in Fig. 2 by the rod A which has a little metal ring slipped loosely over it, through which R passes. Before a reading is to be taken, the switches are set as shown in the figure and R is connected by a string to the specimen in the search coil. Then the

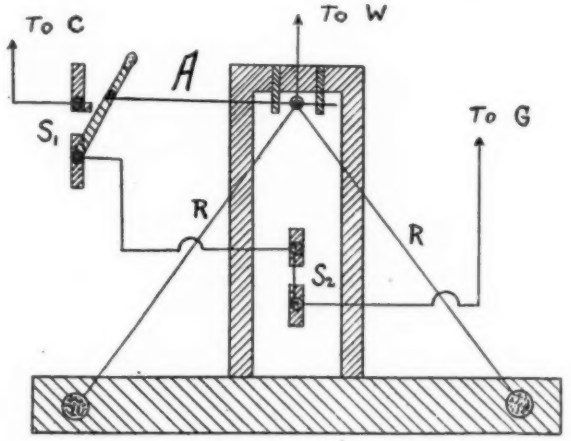

Fig. 2. Circuit controlling device.

operator closes S_1, A slides ahead and releases the ring retaining R in place, and R moves back toward S_2 and pulls the specimen out of the search coil. When R reaches S_2 it catches the handle (not shown in the figure) and opens the switch so that the circuit is again open. The length

of the time interval during which the circuit is closed can be varied by varying the position of S_2. Devices of a similar nature have been used by Wwedensky,[3] Gildemeister[4] and Wilson.[5]

In order to test the validity of the method and at the same time calibrate the set-up, the rod A in Fig. 2 was connected to the handle of a switch which was inserted in the circuit containing the magnetizing coil, so that small changes in the field through the search coil could be made and the corresponding deflections read. The deflections proved to be proportional to the deflecting currents and to be independent, within reasonable limits, of the tension on R and the position of S_2.

The deposits were made in a device described by Steinberg,[6] provided with a few minor changes, on tin or aluminum foil approximately 7.5 by 2.5 cm. The weights of the deposits were determined by weighing the foil before and after deposit, and the thicknesses were computed assuming the value of density in bulk. After this had been done, the foil was rolled upon a thin thread so as to form a uniform cylinder, 7.5 cm long and thin enough to be inserted in the search coil.

The iron used in this investigation was a sample of Armco iron, which, according to an analysis furnished by the makers (Page Steel and Wire Co., Bridgeport, Conn.) is 99.874 per cent pure. The nickel was furnished by Baker & Co. and contained, according to an accompanying analysis, 0.27 per cent iron, 0.05 per cent copper, and 0.004 per cent manganese. Cobalt wire was made by electrolytic deposition on a one mil tungsten wire by a method described by Langbein and Brannt in "Electrodeposition of Metals." These deposits presented a very bright surface but were exceedingly brittle so that great care had to be taken not to put any strain on them in inserting them in the evaporation device.

Experimental Results

In Table I the results obtained for the three metals will be found. The following symbols are used:

δ = thickness of film in millimicrons ($m\mu$);
d_m = deflection for $H = 139$ gauss;
B_m = magnetic induction for $H = 139$ gauss;
B_r = remanence;
H_c = coercive force for a cycle in which the maximum value of the field was 139 gauss.

[3] Wwedensky, Ann. der Phys. **66,** 110 (1921)
[4] Gildemeister, Ann. der Phys. **23,** 401 (1907)
[5] Wilson, Phys. Soc. Proc. **34,** 55 (1922)
[6] Steinberg, loc. cit.[1]

TABLE I. Experimental data.

Experimental data for iron, nickel and cobalt films.

	δ	d_m	B_m	B_r	H_c
Iron	14	16	16800	87 per cent	111
	26	34	19000	88	116
	32	45	20400	88	91
	39	66	23800	91	96
	40	63	22900		114
	55	78	22100	83	99
	59	108	26000	89	39
	74	125	24400	88	45
	98	148	22000	93	53
	150	222	21500	90	43
Nickel	51	6	1725	33 per cent	
	61	6	1460		105
	93	16	2530	55	77
	194	37	2710	54	75
	207	51	3280	59	56
	215	43	2830	53	61
	268	57	3070	56	62
	295	60	2890	58	55
Cobalt	18	22	16100	74 per cent	89
	22	32	19200	83	75
	25	33	17900	76	83
	36	40	14400	88	73
	63	78	16200	76	83
	77	91	16100	62	34

It will be seen that for each metal the films can be divided into two groups according to the coercive force. Fig. 3 represents the hysteresis curves obtained for one specimen from each group of iron films.

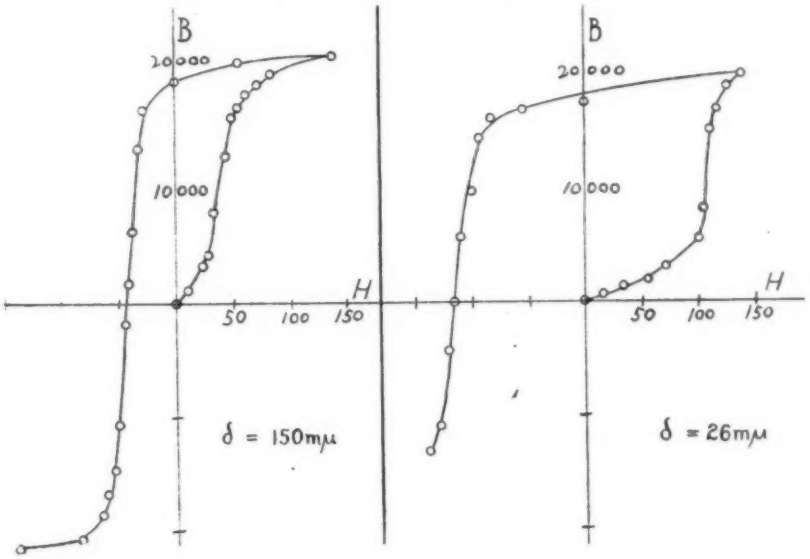

Fig. 3. Hysteresis curves for iron films.

It will be noticed that the thinner of the two films does not give a symmetric hysteresis curve. This is due to the fact that the maximum value of the field was not far enough beyond the coercive force to produce symmetry.

The lapse of time had very little influence on the magnetic properties of the films. While a slight decrease in permeability could be detected in the iron films four weeks after the first test was made, no change was detected in the remanence or coercive force of any of the three metals.

All of the cobalt films and some of the iron films acquired a magnetic moment during the process of deposition, which in some of the films amounted to as much as one-fourth of the moment due to the maximum field used in the tests, $H = 139$ gauss.

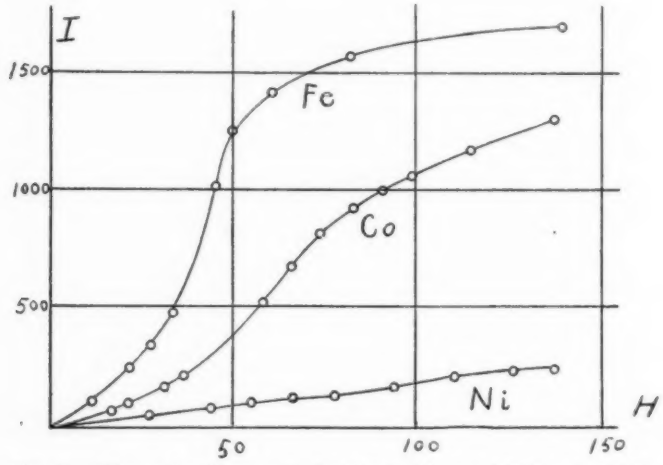

Fig. 4. Magnetization curves for films of iron, nickel and cobalt.

Discussion of the Results

In Table II and Fig. 4 are exhibited the most important results obtained.

Table II

Collected results for all metals.

	I_m	I_r	H_c
Fe.....	1700	90 per cent	43
Ni.....	234	56	62
Co.....	1300	62	34

The values in the table are taken from the thicker groups of films in Table I. I_m and I_r are the maximum and the remanent values, respectively, of the intensity of magnetization.

In regard to the maximum value of I_m, the iron films appear not to be different from bulk iron. In the case of nickel I_m falls considerably below the accepted value for bulk metal (400, according to the Landolt-Börnstein tables). This is probably due, at least in part, to the very slow increase of I with the field. It may also be due, in part, to oxidation of the metal at the high temperature, which manifested itself during the process of deposition in a dull surface without metallic luster and which is largely responsible for the all too great irregularities in the B_m column of Table I.

In the cobalt films I_m reaches considerably higher values than are ordinarily given for bulk metal. Thus, Ewing[7] gives $I=800$ for $H=140$; Fleming, Ashton and Tomlinson[8] give 650 for $H=114$; Rowland[9] gives a still lower value. The samples used in these cases, however, contained considerable amounts of impurities. It therefore appears that cobalt is capable of a higher intensity of magnetization than has been heretofore attained. Moreover, the curve for cobalt in Fig. 4 indicates that still higher values might be obtained if stronger fields were applied.

In connection with the above results it should be noted that in work on thin films of electrolytic iron I_m has in all cases been found lower than for bulk iron.[10] Leick[10] investigated nickel and cobalt as well as iron and found for both approximately the same value as given for nickel in Table II. This low value for cobalt Leick explains as due to sponginess of the deposits.

The most remarkable result in Table II is the high value of the coercive force for all three metals. An explanation of this may perhaps be looked for in the crystalline structure of the films. It has been shown[11] that the size of the crystal granules in the metal has great influence on the specific resistance and the coercive force of iron but that the remanence and the maximum value of the magnetization are practically unaffected by changes in crystal size. Over a considerable range of crystal size the resistivity and the coercive force were both found to be linear functions of the number of crystals per unit length.

There is evidence[1] that the films are crystalline, but that the crystal granules are very small. Thus, Steinberg found evidence of crystal structure in iron films by an x-ray method, but failed to find any such

[7] Ewing, Magnetic Induction in Iron and Other Metals.
[8] Fleming, Ashton and Tomlinson, Phil. Mag. **48,** 271 (1899)
[9] Rowland, Phil. Mag. **48,** 271 (1874)
[10] Kaufmann and Meier, loc. cit.[2];
Leick, Ann. der Phys. **58,** 691 (1896);
Schild, Ann. der Phys. **25,** 586 (1908)
[11] Thompson, Phil. Mag. **31,** 357 (1916)

structure under the microscope. This fact, together with the very high specific resistance of the films,[1] indicates that the crystal granules are very small. Therefore, according to the investigation of Thompson cited above, we should expect to find a high value for the coercive force.

The abrupt change in the coercive force at a certain thickness of film recalls the work of Maurain,[12] who found that when iron was deposited in a constant magnetic field, the magnetic moment of the deposit did not increase in proportion to the amount deposited until the thickness of the deposit was about 80 mμ. For nickel the same was found to be true, only in this case the critical thickness was 200 mμ. On the molecular field theory this is taken to mean that at those thicknesses the molecular field reaches a constant value, or, in other words, at those thicknesses the magnetic properties of the metal become definite. The present results may be taken to be in support of this theory in so far as a definite change is here found at a certain thickness, in one of the magnetic properties, namely,the coercivity. Since, however, the other magnetic properties do not change, the evidence is far from conclusive.

Assuming that the size of the crystals in the films is different from the size of those in bulk metal, the conclusions to be drawn are

(1) The maximum intensity of magnetization is independent of the size of the crystals and also of the thickness.

(2) The remanence depends on the crystal size but not on the thickness.

(3) The coercive force depends on the crystal size and also, in the way indicated above, on the thickness.

The above conclusions agree with the work of Thompson cited above, except in the case of the remanence, in which Thompson found no variation. This may have been due to the facts (1) that the crystals in his samples were very much larger than those in the films, and (2) that the range of sizes that he investigated was relatively small. It is interesting to note that the recent work on permalloy[13] and silicon steel[14] bears out the above conclusions.

On the basis of the results of an investigation[15] in which Kundt proved the Hall e.m.f. in ferromagnetic metals to be proportional to the rotation of the plane of polarization, and the results of an investigation by Du Bois,[16] in which the magnetic rotation of thin films of ferromagnetic metals was shown to be proportional to the intensity of magnetization

[12] Maurain, Jour. de Phys. **10,** 123 (1901)
[13] Arnold and Elmen, Bell Technical Journal, July 1923.
[14] Yensen, Am. Inst. Elec. Eng., May and June, 1924.
[15] Kundt, Ann. der Phys. **49,** 257 (1893)
[16] Du Bois, Ann. der Phys. **31,** 941 (1887)

of the metals in bulk, Kundt drew the conclusion that the Hall e.m.f. is also proportional to the intensity of magnetization. If the Hall e.m.f. is plotted against the external field into which the experimental material is introduced, the curve rises rapidly at first as the field increases to a value roughly equal to 4π times the maximum value of the intensity of magnetization of the bulk metal. From this point on the curve rises only slightly as the field is further increased.

A. W. Smith[17] confirmed Kundt's results; however, in extending the investigation to higher temperatures he found the permeability and the Hall e.m.f. to vary in different ways with the temperature. Hence, the Hall e.m.f. cannot be expressed at various temperatures in terms of the permeability alone.

Steinberg[1] studied the Hall effect in thin films of evaporated iron and, in plotting the Hall e.m.f. against the external field, obtained a curve of the same general shape as those obtained by Kundt and Smith. However, the bend in the curve occurred when the external field had reached a value of 10,000 gauss, and the curve rose only slightly as the field was further increased. It was therefore inferred that the maximum value of the intensity of magnetization in the films was given, approximately, by

$$4\pi I_m = 10{,}000$$

so that I_m would be only about one-half of the value for bulk iron.

It will be seen that the present results do not support this conclusion. However, it must be borne in mind that in the study of the Hall effect the field is at right angles to the plane of the film, while the magnetic properties were measured in a direction parallel to it. Until the magnetic properties of the films have been studied at right angles to the film, the question can hardly be regarded as settled. However, until then the present results at least leave a doubt as to the proportionality between the Hall e.m.f. and the intensity of magnetization, even at room temperature.

In conclusion, the writer wishes to thank the members of the Department of Physics at the State University of Iowa for their assistance and interest, and especially Professor G. W. Stewart, under whose direction the work was carried on.

STATE UNIVERSITY OF IOWA,
August 7 1924.

[17] A. W. Smith, Phys. Rev. **30**, 1 (1910)

THE THERMIONIC WORK FUNCTION OF OXIDE COATED PLATINUM

By C. Davisson and L. H. Germer

Abstract

Measurements of the thermionic work function of pure platinum coated with oxides of barium and strontium have been made simultaneously by two methods for the same segment of a uniformly heated filament. The theory of the measurements and the experimental arrangements are the same as used in an earlier experiment on the thermionic work function of pure tungsten. Filament temperatures accurate to $\pm 5°$, were found from the resistance of the filament at 0°C in conjunction with the temperature coefficients of resistance. (1) In the *calorimetric method* the equivalent voltage of the work function was computed from the sudden voltage change resulting from switching off the space current, due to the cooling effect of the emission. The determination was much more difficult than in the case of the tungsten filament, and measurements were made at the single temperature 1064°K. At this temperature the work function φ was found to be equal to $1.79 \pm .03$ volts. (2) In the *temperature variation* method it was found that, after the temperature had been changed suddenly from one value to another, the emission changed approximately exponentially from an initial value to a final steady value. The half value period of this change varied from a few seconds at high temperature to over a quarter of an hour at low temperature. Interpreting this phenomenon as due to a progressive and reversible change of the character of the filament with temperature, the initial emissions after temperature changes from 1064°K were used to determine the b constant of Richardson's equation corresponding to the equilibrium character of the filament at 1064°K, and similar measurements were made for the b constant corresponding to the character of the filament at 911°K. The two determinations lead, through the relationship $\varphi = bk/e$, to 1.79 volts and 1.60 volts for the corresponding values of φ. For 1064°K, then, the two methods give values for φ in agreement. The measurements are, however, not sufficiently accurate to give any indication whether or not an electron within the metal possesses the thermal energy $3kT/2$. The various corrections made and possible errors are thoroughly discussed. It is pointed out that if the transition from the equilibrium state at one temperature to that at another had occurred so rapidly as to avoid observation, a disagreement of 25 per cent between the values of φ given by the two methods would have been obtained which might have been misinterpreted.

IN a previous paper[1] the authors have given the results of experiments undertaken to test whether the value obtained for the thermionic work-function of a metallic filament from observations on the "cooling effect", is, or is not, in agreement with the value of the same function as deduced on thermodynamic principles from the variation of the emission from the filament with temperature. The observations were made on a single

[1] Davisson and Germer, Phys. Rev. **20**, 300 (1922)

filament of pure tungsten. It was found that the independent methods led to values of the work-function which agreed within the probable error of the measurements (about 1 per cent) when the calculations were carried out on the assumption that the conduction electrons in tungsten do not participate in the equipartition of thermal energy, but that a discrepancy of 2.7 per cent appeared between the values when calculated on the assumption that the conduction electrons throughout the metal possess an average kinetic energy equal to $3kT/2$.

In the present paper the results are given of similar measurements made upon a filament of pure platinum coated with the oxides of barium and strontium. The thermionic filament of this type is a complex chemical system changing in character with temperature, with minute variations in the residual gas in equilibrium with it, and with the flow of electrons from it. It is by no means certain, therefore, that the variation of emission with temperature for such a filament will afford a basis for correctly calculating its work function; and, in fact, the results of our measurements show that under certain circumstances the value of the work function estimated from the temperature variation of emission may be entirely incorrect.

The methods employed in these measurements are similar in essential respects to those employed in the measurements on tungsten, and are not given in detail in this paper.

The Filament and Tube

The filament core was a wire of pure platinum .0152 cm in diameter. The potential leads and the lead for returning space current to the middle point of the filament were welded to the core before coating. These leads were also of pure platinum, and were .0025 cm in diameter. The distances between adjacent leads were initially 3.780 cm and 3.787 cm. At the end of the observations the tube was cut open and the total distance between the potential leads was remeasured. This distance was found to have increased from 7.567 cm to 7.617 cm. This increase resulted from the slight tension applied to the filament to keep it taut.

The coating process consisted in applying to the core approximately equal parts of the carbonates of barium and strontium in solid paraffin as a vehicle. The filament temperature was raised sufficiently during the application to melt the paraffin that was brought into contact with it. A subsequent baking in air at about 1000°C drove off the paraffin and reduced the carbonates to oxides. The coating obtained by this process is firm and adherent. Its weight per square centimeter as determined at the conclusion of the measurements was approximately one milligram.

The filament was mounted in a structure similar to that employed in the measurements on tungsten, except that the plates and electrostatic shields were made of nickel instead of molybdenum.

The complete unit sealed to the pumps comprised the main experimental tube, an ionization manometer and a tube containing cocoanut charcoal. During exhaust, which continued over a period of three days, the entire tube was repeatedly baked out at 410°C, with the charcoal tube carried fifty degrees higher. These bakings were alternated—except for the charcoal tube which was heated continuously—with intense heating of the metal parts by electron bombardment and by high frequency currents. The unit was finally sealed from the pumps at a pressure of 10^{-5} mm of mercury, with all filaments glowing and the charcoal at 460°C. On allowing the charcoal to cool to room temperature the pressure fell to 10^{-7} mm, and on cooling it with liquid air the pressure was lower than could be measured with the available equipment which had a limiting sensitivity of about 10^{-8} mm. Liquid air was kept on the charcoal during the entire period of the measurements, and at no time was the pressure measurable.

Calorimetric Measurement of φ

The calorimetric value of φ, the equivalent voltage of the work function, was determined, as in the case of tungsten, from the relation

$$\varphi=\left(\frac{2EI}{E-IdE/dI}\right)\frac{\Delta E}{i}.$$

In this relation ΔE is the change in voltage across the filament which is observed when the heating current, I amperes, is maintained constant and the plate voltage is switched from a negative to a sufficiently positive value to cause saturation of the emission current, i amperes, from the filament. More exactly, ΔE is the change resulting from the "cooling effect" of the emission alone. The observed value of ΔE is made up of this "true" voltage change plus several other "spurious" changes. These spurious changes though important are easily separable from the true effect.

Calorimetric measurements were made at one value of heating current only. This current was 1.00954 amp., corresponding to a filament temperature of 1064° K. The coefficient, $Q=2EI/(E-IdE/dI)$, corresponding to this value of I was found from a short accurately determined E-I curve for the filament, including the standard value of I at its middle point. The value so determined was $Q=-4.19\pm.02$ amp.

It was found that when the plates were switched to a positive potential the emission was never immediately steady, but decreased from an

initial value, rapidly at first, and then more slowly, approaching a final value asymptotically. This behavior made it impossible to measure accurately the ratio $\Delta E/i$ corresponding to the switching on of the plate voltage. All observations were accordingly made by first establishing the positive plate potential and allowing the emission to become perfectly steady. The space current was then measured and the corresponding value of ΔE was found when the space voltage was switched off. The results of one such observation are shown in Fig. 1. From time zero to 10 min. the system was in an essentially steady state. At 10 min. the space voltage was switched off, and the heating current immediately readjusted to its standard value. The initial increase in E is due to the

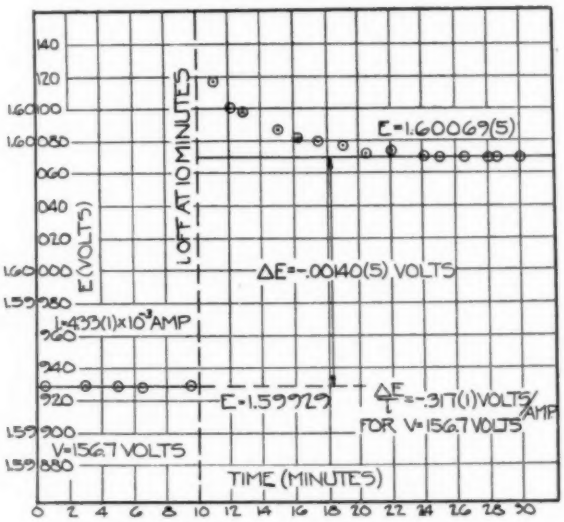

Fig. 1. Readings for determining ΔE.

cessation of the cooling effect of the emission (and to an effect arising from the asymmetry of the emission). The gradual decrease in E to a final steady value is due to a corresponding decrease in the thermal radiation from the plates to the filament.

We require the sudden voltage change which occurs when the space current is switched off. As this could be determined from the data of Fig. 1 only roughly by an extrapolation process, it was necessary to obtain the accurate value of the change in a less direct way. The difference between the steady state values of E (time $<$ 10 min. and time $>$ 24 min.) can be easily determined. The value of $\Delta E/i$ corresponding to this value of ΔE is plotted against the plate potential, 156.7 volts, in Fig. 2. Points obtained in this way for different plate potentials should

theoretically give a straight line whose intersection with the vertical axis determines the value of $\Delta E/i$ corresponding to the initial sudden change in E when the plate voltage is removed.

In all the observations of Fig. 2, with the exception of those indicated by the points A and B, two-thirds of the space current was returned to the middle point of the filament and one-sixth to each end. This is the critical arrangement, as shown in the previous paper, for balancing out

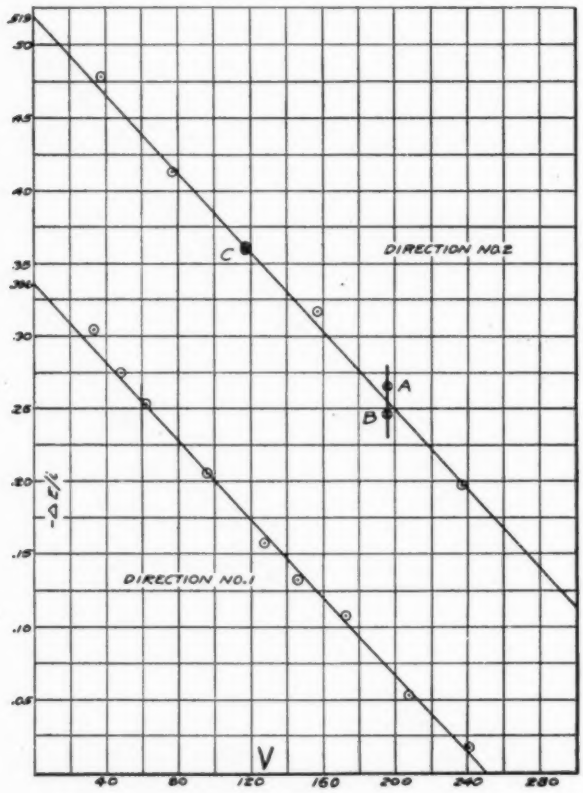

Fig 2. $\Delta E/i$ as a function of plate potential V.

a spurious voltage change that otherwise results from what we have termed the "temperature shift." The point A was obtained with one-half of the space current returned to each end of the filament, and the point B with the whole of the space current returned to the middle point. The separation of these points is 0.0195 volts/amp. while the theoretical separation, calculated from formulas given previously, is 0.017 volts/amp. The agreement with the theory is better than the accuracy of the measurements would lead one to expect.

If equal elements of the filament contributed equally to the emission the points in Fig. 2 would fall on a single straight line, at least for values of V which are great compared with the potential drop along the filament, and the value of $\Delta E/i$ free from effects arising from "back heating" would be given by the intercept of this line with the axis of ordinates. When equal elements do not contribute equally to the emission, the points fall on two parallel straight lines corresponding to the two directions of heating current, as in Fig. 2, and the true $\Delta E/i$ is given to a very close approximation by the average of the intercepts of these two lines. As the intercepts in Fig. 2 occur at 0.336 and 0.519, we have for the final value

$$-\Delta E/i = 0.427 \text{ volts per ampere.}$$

Combining this with the value of Q, we find

$$\varphi = Q\Delta E/i = (-4.19)(-0.427) = 1.79 \text{ volts.}$$

Consideration of Corrections and Errors

The effects of "back heating" and of the asymmetry of the emission are much more marked in these observations than in the previous measurements on tungsten. It seems desirable, therefore, to look rather carefully into the possible errors arising from these and one other cause.

In drawing parallel lines through the points in Fig. 2 the three points for values of V less than 60 volts have been disregarded. When the potential difference between the plates and the negative end of the filament is sensibly greater than that between the plates and the positive end, the emission from the negative end is favored, and the observed values of $\Delta E/i$ are in error by being too great. This effect is noticeable below $V = 60$ volts, but is apparently unimportant above this voltage.

It must also be pointed out that, in drawing straight lines through the points beyond 60 volts, we are assuming that, if φ is not independent of V, the changes in φ are proportional to the changes in V. The emission from a thermionic filament never exhibits perfect saturation; the emission always increases to some extent with the strength of the field at the surface of the filament. The chief cause of this, and perhaps the only cause, is a reduction in the effective value of φ. Thus in Fig. 2 the points representing $\Delta E/i$ would fall on a descending curve even if there were no "back heating." The observed decrease of $\Delta E/i$ with increasing V is the sum of these two effects. In extrapolating along a straight line to $V = 0$ we are assuming that $d\varphi/dV$ is constant for all values of V. That this coefficient is sensibly constant over the range $V = 60$ to $V = 240$ has been verified by plotting the values for log i against V. The points fall very close to a straight line, and from this line we calculate $d\varphi/dV = -8.5$

$\times 10^{-5}$. If the relation between φ and V is non-linear below $V=60$ volts, the extrapolation leads to an incorrect value of φ for $V=0$. This incorrect value may still be used, however, with the value found for $d\varphi/dV$ to obtain correct values of φ in the range of V above 60 volts. Thus for $V=100$ volts, $\varphi=1.79-0.0085=1.78$ volts. Within the limit of accuracy of the measurements, variations of this magnitude are unimportant, and are neglected in what follows.

It is somewhat reassuring to find that the slope of the lines in Fig. 2 is in accord with the slope of a similar line found for tungsten. From considerations appearing in the previous paper it may be shown that the slope of these lines should be given by

$$\frac{d}{dV}\left(\frac{\Delta E}{i}\right)=-\frac{\lambda m}{Q}$$

where λ is the absorptive power of the filament for the low temperature plate radiation, and m is the fraction of this radiation intercepted by the filament. From Fig. 2 $d(\Delta E/i)/dV=.0013$ amp.$^{-1}$. The corresponding coefficient for the tungsten filament was .00043 amp.$^{-1}$. If we indicate by subscripts $_1$ the quantities associated with the tungsten filament and by $_2$ those associated with the coated filament, we have for the ratio of the absorptive powers

$$\frac{\lambda_2}{\lambda_1}=\frac{.0013}{.00043}\,\frac{Q_2 m_1}{Q_1 m_2}.$$

$Q_2=-4.19$, $Q_1=-2.4$ and m_1/m_2 is equal to the ratio of the filament diameters which is 0.5, so that $\lambda_2/\lambda_1=2.6$. If λ_1 is 0.3, as previously estimated, λ_2 comes out 0.78, which is an acceptable value for this constant. The only uncertainty in the "back heating" correction appears to be that involved in locating the parallel lines in Fig. 2.

The extrapolated values of $\Delta E/i$ differ by 40 per cent for the two directions of heating current. This is known to be due to the fact that the emission from the lower half of the filament was somewhat greater than that from the upper half. The returned space current thus on the whole flowed downward in the filament regardless of the direction of the heating current, and gave rise to a ΔE_1 that added to the true effect in one case and a ΔE_2 that subtracted in the other. If ΔE_1 and ΔE_2 are equal, errors due to this cause are, of course, completely avoided by averaging. As far as first order quantities are concerned this condition is satisfied. It seems worth while, however, to find out something about the magnitude of second order quantities. If for the time being we disregard all other effects including the "temperature shift," and consider

only effects arising from the distribution of heating current in the filament we have for the drop per cm along the filament

$$dE_1/dx=\rho I+\rho' j+\tfrac{1}{2}j^2 d\rho'/dI$$

where ρ is resistance per unit length for heating current I, $\rho'=(\rho+Id\rho/dI)$, and j is the portion of the returned space current flowing in the filament at the point x. For the total drop along the filament

$$E_1=2\rho Il+\rho'\int_{-l}^{+l} j\,dx+\frac{1}{2}\frac{d\rho'}{dI}\int_{-l}^{+l} j^2 dx.$$

The total filament length between potential leads is $2l$ and distance is measured from the mid-point. For the main current in the reverse direction, the returned space current j remaining everywhere unaltered, we have

$$E_2=2\rho Il-\rho'\int_{-l}^{+l} j\,dx+\frac{1}{2}\frac{d\rho'}{dI}\int_{-l}^{+l} j^2 dx\,.$$

The average of these is

$$\frac{1}{2}(E_1+E_2)=E+\frac{1}{2}\frac{d\rho'}{dI}\int_{-l}^{+l} j^2 dx$$

where $E=2\rho Il$ is the drop across the filament with no space current flowing. There is, therefore, a second order voltage change

$$\Delta E'=\frac{1}{2}(E_1+E_2)-E=\frac{1}{2}\frac{d\rho'}{dI}\int_{-l}^{+l} j^2 dx$$

which is not zero even for uniform emission.

The voltage change due to the cooling is $\Delta E=\varphi i/Q$, so that, the error introduced by the effect under consideration is

$$\frac{\Delta E'}{\Delta E}=\frac{\Delta\varphi}{\varphi}=\frac{Q}{2\varphi i}\frac{d\rho'}{dI}\int_{-l}^{+l} j^2 dx\,.$$

If the space current is returned equally to the two ends of the filament and the emission is uniform, $j=-ix/2l$ and

$$\frac{\Delta\varphi}{\varphi}=\frac{Qil}{12\varphi}\frac{d\rho'}{dI}\,.$$

If the emission is zero from one-half of the filament and uniform from the other half, $j=i/2$ from $-l$ to 0, and $\frac{1}{2}i\ (1-2\ x/l)$ from 0 to $+l$. Integration gives for this case

$$\frac{\Delta\varphi}{\varphi}=\frac{Qil}{6\varphi}\ \frac{d\rho'}{dI}\ .$$

The error in the present measurements is probably between these limits. $d\rho'/dI$ is approximately 0.19 ohms/amp. and $i=4\times10^{-3}$ amp. Substituting these values and for the other quantities the values already given, we find for the maximum error

$$\Delta\varphi/\varphi=-1.1\times10^{-3}\ .$$

The error due to second order effects is thus not more than one part in a thousand.

Another consideration should be mentioned in this connection. It is assumed that the portions of the space current returned to the two ends of the filament flow into the filament at these points, and that no part of the current flows from one end to the other through the main heating circuit. If the system were perfectly symmetrical this condition would be satisfied. The system was not symmetrical, however, on account of the assymetry of the emission, and therefore the condition was not satisfied. Although errors arising from this cause disappear to the first order on averaging it was thought worth while to test for the existence of this effect. For this purpose the two points C (Fig. 2) were taken, one with a 6-volt storage battery in the main heating circuit, the other with a 12-volt battery and additional resistance. The fact that doubling the heating circuit resistance leaves the value of $\Delta E/i$ unaltered shows that even the first order effect is unimportant.

There is one other source of error which is associated with the grid action of the shielding boxes in limiting the emission from the filament for a short distance out from each box. The precautions against this error consisted in maintaining each box 3.00 volts negative to the filament at its point of emergence. It is doubtful if this arrangement completely eliminates the error involved, but we are reasonably certain from theoretical considerations and from auxiliary experiments made in connection with the measurements on tungsten that the residual error is not greater than one percent, and that it is probably much less.

Assuming that all errors of importance have been considered, and allowing rather generously for each, we believe that we are justified in writing

$$\varphi=1.79\pm.03 \text{ volts}$$

for this particular oxide coated filament.

The Temperature-Emission Relation

A difficulty not met with in the calculation of φ from the temperature variation of emission in the case of tungsten presented itself in the present measurements. It was found that when the emission was steady at a particular temperature, it was not immediately steady on changing the filament temperature suddenly to a different value. On increasing the temperature the emission changed from an initial value to a lower steady value. On decreasing the temperature the change was in the opposite direction. The half value period of these changes, which were approximately exponential, varied from a few seconds at the highest temperatures to more than fifteen minutes at the lowest. We may thus think of two distinct modes of variation of emission with temperature, the variation of the steady value of emission, and the variation of the initial value. Further, the initial value of emission for a particular temperature was not unique but depended on the previous temperature at which the

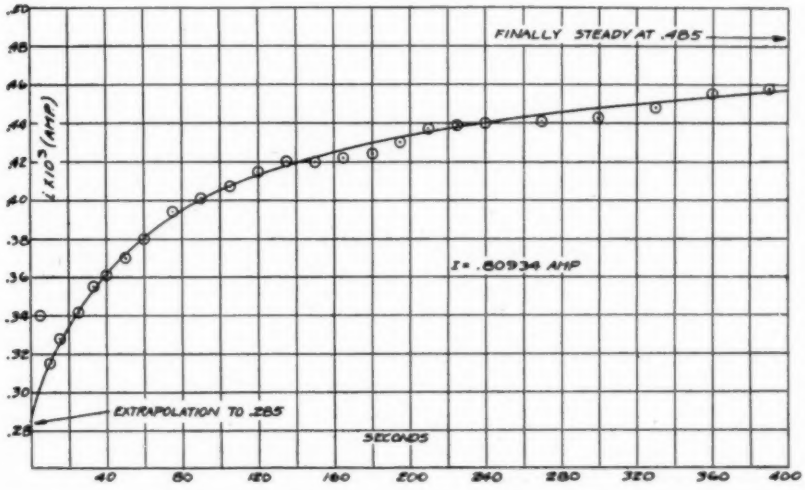

Fig. 3. Variation of emission with time after sudden change of heating current.

emission had been allowed to become steady. Under the circumstances the proper procedure appeared to be to allow the emission to become steady at the temperature at which the cooling measurements had been made (1064°K) and to observe both initial and final values of emission on changing from this to another temperature. An example of such a determination is shown in Fig. 3. With the heating current at 1.00954 amperes and the emission steady the current was changed suddenly to 0.80934 amperes. The curve of emission against time was extrapolated, as indicated, to find the initial emission for this new temperature, and

the final steady value was found by continuing observation until the emission was sensibly constant.

The filament temperatures corresponding to the various heating currents were found from the filament resistances by means of the relation

$$R_t = 0.4217(1 + .003946t - 5.85 \times 10^{-7} t^2) .$$

The temperature coefficients in this formula were determined at the Bureau of Standards for two samples of the wire used as filament core, and R_0 ($=0.4217$ ohms) was evaluated from determinations of R_t made at room temperature. These room temperature measurements were made at the conclusion of all other observations and after the tube had been "let down to air." A calibrated mercury thermometer inserted into the tube through an opening in the top was used in determining the temperature of the filament.

The results of the temperature-emission measurements are given in Table I. The numerals in the first column indicate the order in which the observations were made.

TABLE I

Temperature-emission data for oxide coated filament.

Standard temp. 1064°K. Filament area = 0.33 cm²

Order	I (amp.)	resistance (ohms)	Temp.	Initial i (amp. $\times 10^3$)	Final i (amp. $\times 10^3$)
6	.66711	1.2889	842°K	.02046	.....
4	.70144	1.3224	867	.040	.....
8	.76674	1.3838	912	.1282	.277
2	.80934	1.4218	940	.2772	.472
9	.86047	1.4653	973	.551	.878
1	.91895	1.5138	1010	.....	1.64
7	.96327	1.5485	1036	2.258	2.499
0	1.00954	1.5836	1064	3.786	3.79
10	1.06254	1.6252	1097	.....	5.95
3	1.11941	1.6660	1130	10.10	8.99
5	1.16371	1.6973	1155	.....	12.00

Two small corrections have been applied to the directly determined values of emission in arriving at the values appearing in columns 5 and 6. In the first place, all space currents were measured with an effective plate potential of approximately 100 volts. Each determination was then corrected to correspond to $V = 100$ volts by means of the independently determined relation between i and V at the particular temperature of the point. In the second place, the steady emission at the standard point immediately before the heating current was changed to a new value, usually differed somewhat from 3.786×10^{-3} amp. which had been chosen

as the standard value for this quantity. To allow for these variations the values of emission at the various temperatures were multiplied by the ratio of the standard to the actual emission at the standard temperature. Over the period of four days during which these observations were made, this ratio varied between the limits 1.03 and 0.969.

At the highest temperature, the rapidity of the change of emission made the determination of its initial value uncertain, while at the lowest temperatures determination of the final steady value became impractical on account of the slowness of the change. These omissions are indicated by the spaces in the table.

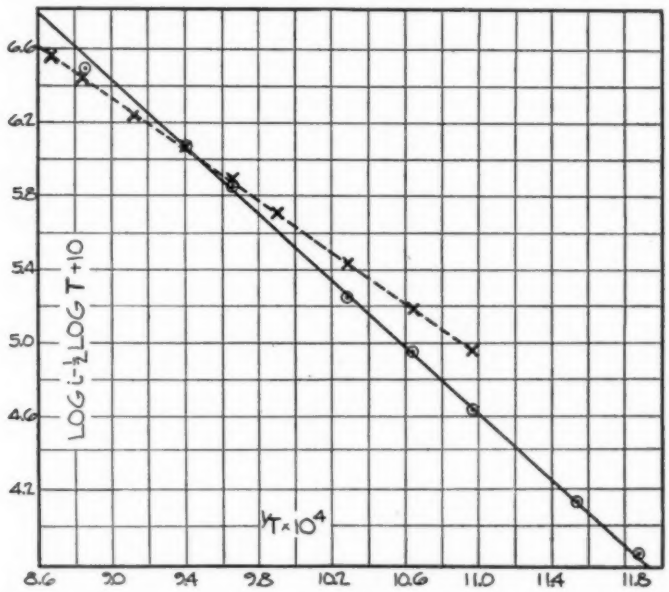

Fig. 4. $\text{Log}_{10}\, i - \frac{1}{2}\log_{10} T$ as a function of $1/T$ for initial emission (full line) and final emission (dashed line).

In Fig. 4, $(\log_{10} i - \frac{1}{2}\log_{10} T)$ is plotted against T^{-1} for both initial and final values of emission, the former being represented by circles, and the latter by crosses. Each set of points falls reasonably well on a straight line, indicating that both initial and final emissions may be represented by Richardson's equation with appropriate constants. The slope of a line on such a diagram is a measure of the quantity $[(\varphi e/k)\log_{10}\epsilon]$. The solid line in Fig. 4 is located with reference to the plotted points, *but has been drawn in with a slope determined solely by the value of φ found in the calorimetric measurements* (1.79 *volts*). It is strikingly evident that the initial values of the emission—not the final values—are directly related

to the calorimetric φ. The work function indicated by the line of final values has an equivalent potential

$$\varphi = 1.38 \text{ volts.}$$

Similar observations were next made at a new "standard temperature" —in this case 911° K. The data obtained in these observations are given in Table II, and are shown graphically in Fig. 5.

TABLE II

Observations for standard temp. 911°K.

Order	I (amp.)	resistance (ohms)	Temp.	Initial i (amp.$\times 10^3$)	Final i (amp.$\times 10^3$)
1	.65746	1.2785	835°K	.0475	....
7	.69645	1.3171	863	.0905	.113
0	.76625	1.3829	911	.300	.300
6	.80050	1.4132	934	.497	.457
2	.86176	1.4660	973	1.13	....
3	.98124	1.5624	1047	3.81	....
4	1.01199	1.5853	1065	5.70	....
5	1.02047	1.5915	1070	5.70	4.08

The final values of emission, as far as they were observed, were in agreement with those of the previous measurements. The initial values determine a line corresponding to $\varphi = 1.60$ volts, that is, to a value 11 percent less than that found at 1064° K. The light solid line in Fig. 5 is that of the initial emissions in the previous measurements. At this lower standard temperature considerably greater time was required for the emission to reach equilibrium. The observations were fewer, and the value of φ is less reliable. There seems no reason for doubting, however, that there is a marked decrease in φ for a decrease of 150 degrees in temperature, nor that a calorimetric determination at this lower temperature would have been found in sensible agreement with 1.60 volts. Although the calorimetric measurements are extremely laborious, it was planned to make other determinations of the calorimetric φ at temperatures above and below the original standard. This project was necessarily abandoned, however, when an accident to the tube rendered further measurements impossible.

CONSIDERATION OF ERRORS

The only important source of error in determining values of φ from the temperature variation of emission is the estimation of filament temperatures. We have assumed that the temperature-resistance properties of the platinum core are unaltered by the coating, and that the coating carries no appreciable part of the heating current. Numerous experiments on the chemical properties of coated filament, carried out in this

laboratory by Dr. J. E. Harris, have established that during the time the filament is glowed in air a certain amount of platinum enters into chemical combination with the oxides of the coating. These reactions are reversed on subsequent glowing in vacuo, but the platinum which was combined with the oxides remains in the coating in a finely divided state. It has also been found by Dr. Harris that barium and strontium,

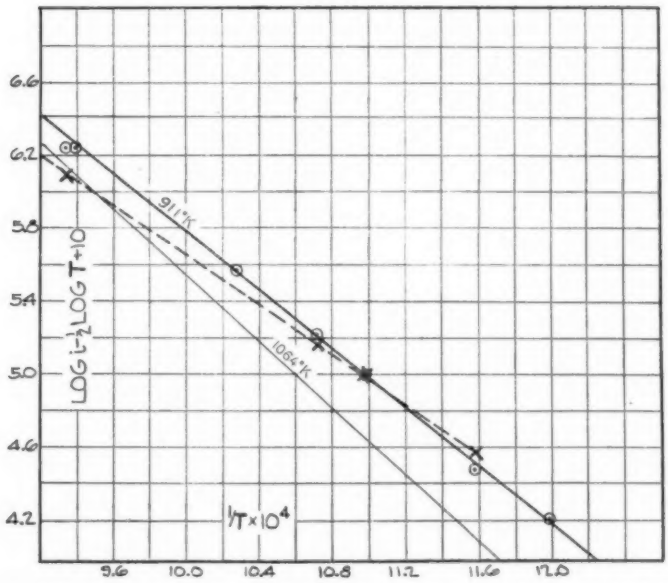

Fig. 5. *Results for standard temp. 911°K. Initial emission given by full line.*

or their oxides, diffuse from the coating into the core of the filament during the time it is used as a source of electron emission. Either of these actions might conceivably vitiate the temperature measurements. We believe, however, for reasons stated below, that such errors were of no great importance.

A determination of the zero resistivity of the core at the conclusion of all other measurements, in which the observed elongation and reduction in cross section of the core were allowed for, gave for this constant a value less than one-half of one percent greater than the value found in another experiment for the same platinum in an uncoated condition. The important thing, of course, is not the constancy of this resistivity but that of the temperature coefficients. It appears, however, from an examination of data on platinum resistance thermometers published by the Bureau of Standards[2] that a fairly definite correlation exists between

[2] Bull. Bur. Stds. **6**, No. 2, p. 156

changes in resistivity and changes in the temperature coefficients. On the basis of this correlation we estimate that our highest computed temperature (1155°K) may be 4 or 5 degrees too low. The error in φ introduced by systematic errors in temperature of this magnitude would be less than one percent.

The second assumption that no part of the heating current is carried by the coating is probably valid enough. We know from experiments performed in this laboratory that at low temperatures the resistance of the coating is extremely high, and that even at high temperatures it is possible to have coated filaments at different potentials in contact with one another without appreciable conduction between them, apart from that due to their thermionic emission.

Discussion of Results

In the measurements on tungsten it was found that the best agreement between the differently determined values of the work function was obtained by supposing that the work involved in the liberation of a single electron is made up of two parts, a part equivalent to a voltage φ_0, representing work done against electrical forces at the surface of the metal, and another part representing work done in giving to the electron the kinetic energy it is known to possess at the time of emission. The value φ_0 should be given by $\varphi_1 - 2kT/e$, where φ_1 is the calorimetric value of φ, or by $\varphi_2 - 3kT/2e$, where φ_2 is the value of φ from the temperature variation of emission. In the present case we have $\varphi_1 = 1.79$ volts, and from Fig. 4 this is apparently the best value for φ_2 for the initial values of emission. We have then from φ_1, $\varphi_0 = 1.79 - 0.18 = 1.61$ volts, since $T = 1064°$K, and from φ_2, $\varphi_0 = 1.79 - 0.14 = 1.65$ volts. The difference between these quantities is certainly not greater than the probable error of the combined measurements, so that the results cannot be taken to afford any positive information in regard to the nature of the work function.

The agreement, on either theory, between the calorimetric φ and that calculated from the variation with temperature of the initial values of emission, is taken to mean that as far as this emission is concerned the thermodynamic specification of the system is essentially complete; that no actions are involved beyond those contemplated in the simple theory of electron emission. It must also be taken to justify, within limits, the assumptions commonly made in regard to the properties of an electron atmosphere and in regard to the relation between rate of emission of electrons and the pressure of an atmosphere of electrons in equilibrium with the filament.

The phenomena exhibited by the line of final values of emission are not so readily disposed of. It is clear that if the transition from initial to final emission had been much more rapid the existence of the line of initial values might never have been suspected. These are the circumstances referred to in the introduction, under which an entirely incorrect value of the work function would be inferred from the temperature variation of the emission. It is not impossible that, in the case of tungsten, the difference between the values of φ obtained on the classical theory by the two methods is to be accounted for in this way. If the transition had been rapid at all temperatures in the present case we would have been confronted with the task of accounting for a 25 percent discrepancy between the two values of φ. Although this difference is not now to be regarded as a discrepancy between measurements of the same quantity, it is still of great importance to consider the possible significance of the line of final values of emission.

An explanation in very general terms appears obvious enough. The filament in equilibrium at the original standard temperature is in a particular condition which may be indicated vaguely as condition C_1. It was for the filament in this condition that the calorimetric φ was determined. On changing the temperature suddenly to a new value the filament is initially still in the condition C_1 and changes only slowly to the condition appropriate to the new temperature. The initial values of emission are then all for the filament in the condition C_1, and the variation of this emission with temperature should be determined by the calorimetric φ for the filament in condition C_1, as is found to be the case. What we really want to know, however, is what constitutes the condition of the filament and in what way does this condition determine the emission. It is hardly possible, of course, that the complete answers to these questions are to be found by making observations of the sort described in this paper. For this reason it seems inadvisable to pursue the matter further at this time.

It must be clearly understood that as far as numerical values are concerned the results given in this paper apply to the particular filament on which the measurements were made. It was stated in the introduction, not without reason, that the oxide coated filament is a complex chemical system—and in this system is included the residual gas in the tube. An unfortunate consequence of this fact is that each coated filament has an individuality that is not quite the same as that of any other coated filament. The variations, however, are those among the individuals of a race. The same characteristics in varying degrees are exhibited by

all. Thus it is that while no great importance can be attached to the constants of any given filament, the study of an individual is not the investigation of an isolated phenomenon.

We take pleasure in thanking Drs. H. D. Arnold and W. Wilson for their sustained interest in these work-function studies.

RESEARCH LABORATORIES OF THE
AMERICAN TELEPHONE AND TELEGRAPH COMPANY,
AND THE WESTERN ELECTRIC COMPANY, INCORPORATED
July 24, 1924.

FORMULAS FOR THE AMPLIFICATION CONSTANT FOR THREE-ELEMENT TUBES IN WHICH THE DIAMETER OF GRID WIRES IS LARGE COMPARED TO THE SPACING

By F. B. Vogdes and Frank R. Elder

Abstract

Previous formulas which have been developed for the amplification constant in plane parallel three element tubes, do not take full account of the diameter of the grid wire. In this paper after making a conformal transformation, from the plane w to the plane z, where $w = \log z$ and making an approximation which is sufficiently accurate for all diameters of grid wire up to 1/3 the spacing, formulas are derived which give results in good agreement with the values determined from two series of actual tubes for which the older formulas are greatly in error or fail entirely.

ONE of the important constants of the three-element tube is its so called amplification constant. It is important to know how this constant, which is usually designated by μ varies with changes in tube design. For certain simple geometric forms, parallel plane and concentric cylindrical elements, the amplification constant is usually calculated as a problem in pure electrostatics. Solutions for these problems have been given by J. Clerk Maxwell[1], M. Abraham,[2] M.v. Laue[3] and R. W. King.[4]

A serious limitation in the applicability of these solutions is due to the necessity of assuming the radius of the grid wire as very small, compared to its spacing, in the derivation of the formula. It has been recognized for some time by those interested in tube design that, in cases where the grid was composed of wires fairly close together, the calculated μ deviated very greatly from the observed value.

It was of interest then to attempt the derivation of a formula which would give a closer approximation to the observed values.

Consider the parallel plane three-element tube, a section through which has been shown in Fig. 1. The grid is composed of wires of radius r spaced at a distance 2π apart. The anode is a plane at distance s from the grid on the one side, while the cathode is a plane at an infinite distance on the other side. In order to avoid end effects, the tube will be considered as infinite in extent.

[1] J. C. Maxwell, Electricity and Magnetism, 3rd ed. Vol. 1 §203
[2] M. Abraham, Arch. f. Elektrotechnik **8,** 42 (1919)
[3] M. v. Laue, Ann. der Phys. **59,** 465 (1919); Jahrb. d. Drahtl. Telegr. **14,** 243 (1919)
[4] R. W. King, Phys. Rev. **15,** 256 (1920)

The plane upon which this figure is drawn will be called the w plane. In order to get a solution of the problem, this figure will be transformed into the z plane by the transformation $w = \log z$. This is well known in the theory of functions as a conformal transformation, and hence the relationship of equipotential surfaces and lines of flux will be preserved by the transformation. In Fig. 2 is shown this transformation. The filament has become the point at O, the grid which is shown as A,B,C,D,E for a series of different radii has become the nearly circular figures marked A',B',C',D',E' about the point 1 and the anode has become a circle about O of radius x. It will be readily seen that the figure E',

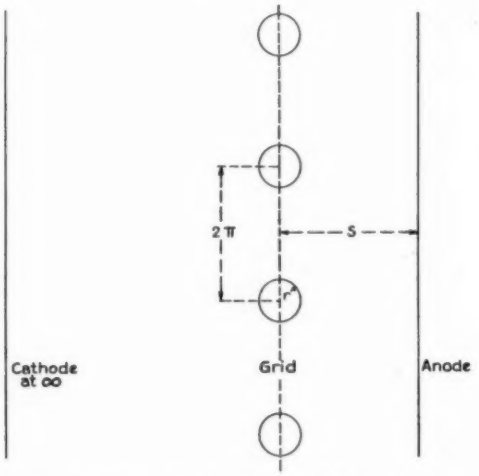

Fig. 1. Section through plane parallel three-element tube.

which corresponds to the circle E of radius $\frac{1}{2}\pi$ of the w plane, departs from a circle by a considerable amount. On the other hand D' and all smaller figures are essentially circular and their effect should be practically that of circles. Now D represents a wire diameter equal to 1/3 the spacing. The formula to be developed will be based on the assumption that these figures are circular and it should hold for relative dimensions of this order of magnitude. Of course the smaller the diameter of the wire the closer is the approximation.

The equipotential surfaces around the figure D' soon become practically circular, and at a distance corresponding to the spacing of the grid wires, the potential has become practically symmetrical about O and the error involved in taking the circle at x to be an equipotential surface representing the anode is negligible. This is more clearly shown in Fig. 3 which represents the equipotential surfaces and lines of flux for the plane

parallel tube as actually constructed from Fig. 2 for the case where the grid wire diameter is 1/3 the spacing. In this case it is easily seen

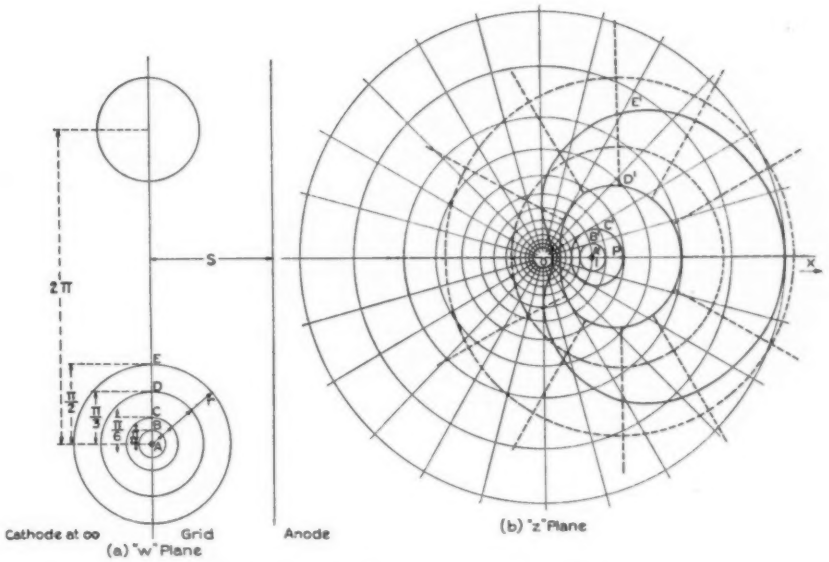

Fig. 2. Conformal transformation from w to z plane.

that even in so short a distance as the diameter of the grid wire, the equipotential surfaces are nearly planes. In tubes as actually constructed these assumptions are usually fulfilled.

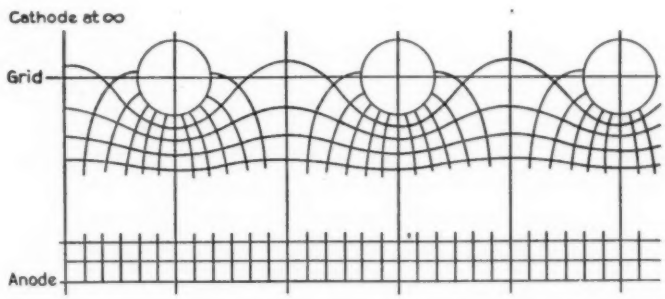

Fig. 3. Equipotential surfaces and lines of flux in plane parallel three-element tube.

In order to more clearly show these relationships part of Fig. 2 has been redrawn as Fig. 4. From the method of transformation we have $d_1=1-e^{-r}$ and $d_2=e^r-1$, where r is the radius of the grid wire. The point P which is the center of the essentially circular figure, is then $1+\frac{1}{2}(d_2-d_1)=\frac{1}{2}(e^r+e^{-r})$, and considering this figure as a circle its radius

is $\frac{1}{2}(e^r-e^{-r})$. If now a charge $-Q$ is placed at P then the potentials V_1, V_2, V_3 of filament, grid and anode become

$$\begin{aligned} V_1 &= C+2\,Q\,\log_e \tfrac{1}{2}(e^r+e^{-r}),\\ V_2 &= C+2\,Q\,\log_e \tfrac{1}{2}(e^r-e^{-r}),\\ V_3 &= C+2\,Q\,\log_e x. \end{aligned} \tag{1}$$

The amplification constant can be defined as $\mu=(V_3-V_1)/(V_1-V_2)$, and for convenience the potential of the grid will be taken as zero, thus

$$C=-2\,Q\,\log_e \tfrac{1}{2}(e^r-e^{-r});$$

hence,

$$\begin{aligned} V_1 &= 2\,Q\,\log_e [(e^r+e^{-r})/(e^r-e^{-r})];\\ V_2 &= 0;\\ V_3 &= 2\,Q\,\log_e [(2x/(e^r-e^{-r})]. \end{aligned} \tag{2}$$

Whence we have

$$\mu=\frac{\log_e x-\log_e \tfrac{1}{2}(e^r+e^{-r})}{\log_e(e^r+e^{-r})-\log_e(e^r-e^{-r})}. \tag{3}$$

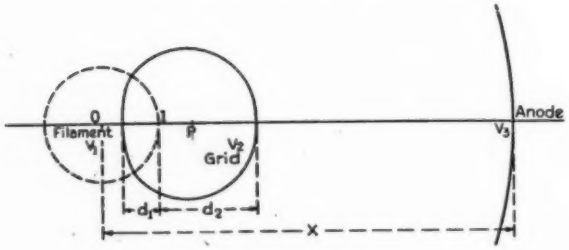

Fig. 4.

Further it is seen that $x=e^s$, and if n is the number of grid wires per centimeter, all dimensions must be multiplied by $2\pi n$ which gives finally

$$\mu=\frac{2\pi ns-\log_e \tfrac{1}{2}(e^{2\pi nr}+e^{-2\pi nr})}{\log_e(e^{2\pi nr}+e^{-2\pi nr})-\log_e(e^{2\pi nr}-e^{-2\pi nr})}. \tag{4}$$

In the limit as r approaches zero this becomes

$$\mu=\frac{2\pi ns}{\log_e(1/2\pi nr)} \tag{5}$$

which is the familiar formula for the plane parallel case.

In order to obtain the corresponding formulas for coaxial cylindrical tubes, it will be sufficient to show that this type of tube can be readily transformed to the plane parallel tube. In Fig. 5(b) is shown a section through such a tube, and this section is supposed to be drawn on the z plane. The filament is a wire at O, the grid consists of n wires per cm, each wire of radius r, equally spaced on a circle of radius r_g, and the

anode a circle of radius r_p about O. This figure is transformed onto the w plane, Fig. 5(a), using the same transformation as before, namely $w=\log z$, and for convenience r_g is taken as the unit. It is readily seen that the distance between grid and anode of the plane parallel tube is given by $S_2-S_1=\log_e r_p-\log_e r_g$ or $S_2-S_1=\log_e(r_p/r_g)$; but as r_g was taken as the unit we have $S=r_g\log_e(r_p/r_g)$. Substituting this value of S in Eq. (4) gives

$$\mu=\frac{2\pi n r_g\log_e(r_p/r_g)-\log_e\frac{1}{2}(e^{2\pi nr}+e^{-2\pi nr})}{\log_e(e^{2\pi nr}+e^{-2\pi nr})-\log_e(e^{2\pi nr}-e^{-2\pi nr})}. \tag{6}$$

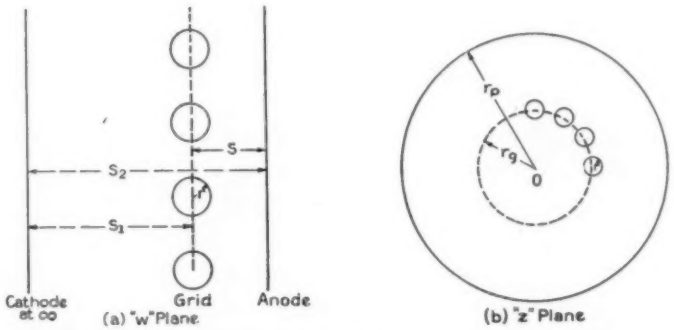

Fig. 5. Transformation of cylindrical to plane parallel tube.

In the limit as r approaches zero this becomes

$$\mu=\frac{2\pi n r_g\log(r_p/r_g)}{\log_e(1/2\pi nr)} \tag{7}$$

which is identical with the formula derived by M. Abraham.[2]

Formulas (4) and (6) are applicable in all cases where the diameter of grid wire is equal to or less than 1/3 the spacing and will give fair approximations even beyond this. Formulas (5) and (7) may be used only in those cases where the grid wire diameter is very small compared to its spacing.

For the so-called inverted tube, in which the anode is a wire of radius r_p in the axis of the tube, the grid has n wires per cm of radius r equally spaced on a circle of radius r_g and the cathode is a cylinder of radius R, the value of S is given by $S=r_g\log_e(r_g/r_p)$. This value of S when substituted in formula (4) should give the amplification constant. No data were available to check this result.

It must be emphasized again that in the solution of such problems as these only conformal transformations should be used. It is recognized

that the formulas developed above are approximations to the true solution, but it seems to us that the basis of the approximation is a reasonable one. While not many data are available for checking these formulas, it will be seen below that they give rather close approximations in cases where the ordinary forms either fail or give absurd results.

Data were available on a series of four groups of tubes with dimensions as follows:

Anode diameter, 1.375″ (3.49 cm);
grid diameter, .857″ at the bottom (2.18 cm), .919″ at the top (2.33 cm);
grid wire diameter, .050″ (0.127 cm);
number of grid wires, 12, 16, 20, 24.

In order to get the grid off the form upon which it was assembled, it was made slightly tapering. Instead of taking an average of the two grid diameters it was thought advisable to calculate the values for the two limits; then the observed value ought to lie between these two values, and depending upon how the experimental value was obtained, it should be closer to one or the other of the two limits. If μ was determined near "cut off" then the observed μ should approach the smaller calculated value, while if the determination was made at points where the plate current was of considerable magnitude, as was actually the case, then the observed μ should move toward the larger calculated value. Calling μ_1 and μ_2 the values calculated for bottom and top dimensions respectively, the following table gives the results obtained by using Eq. (6). For comparison results obtained, by use of Eq. (7) are also given where possible, but the value of μ cannot be calculated for the cases of 20 and 24 grid wires since the term $\log_e(1/2\pi nr)$ becomes negative for a number of grid wires greater than 17.2 and 18.4 for the diameter at the bottom and top respectively.

TABLE I

Calculated and observed values of μ for tubes with tapering grid.

Number of grid wires	12	16	20	24
Calculated using Eq. (6) μ_1	10.6	23	40.4	76.4
Calculated using Eq. (6) μ_2	8.5	17.5	34.2	62
Observed μ	9.3	19	38	72
Calculated using Eq. (7) μ_1	15.7	110.5	....	....
Calculated using Eq. (7) μ_2	11.4	46.4	....	....

The dimensions of a series of three groups of tubes of another type were as follows:

Anode diameter 3/16″ (0.477 cm);
grid diameter .100″ inside;
grid made as a spiral with (a) 32 turns per inch of .003″ wire; (b) 56 turns per inch of .003″ wire, and (c) 56 turns per inch of .005″ wire.

In order to find the value of n for use in the formulas where the grid of the tube is made as a spiral, the total length of wire per cm length of tube is divided by the grid circumference. In this particular type of tube the spiral was supported by a single .020″ wire and some allowance for the effect of this support must be made. As the diameter of this wire is much larger than the grid wire, it seemed that a reasonable approximation would be to regard this effect as equivalent to 1 cm more of grid wire, the larger diameter off-setting the slightly greater distance out and the useless parts at the crossings. Table II gives the results of the calculation.

TABLE II

Calculated and observed values of μ for tubes with spiral grids.

Tube type:	(a)	(b)	(c)
Calculated by Eq. (6):	5.97	16.7	34.5
Calculated by Eq. (7):	6.21	19.6	146.5
Observed:	6.25	18	34

It is thus seen that in the cases where data are available the agreement between observed values and those calculated by the formulas (4) and (6) here developed is very satisfactory.

RESEARCH LABORATORY,
GENERAL ELECTRIC COMPANY,
SCHENECTADY, N. Y.
August 16, 1924

QUANTITATIVE THEORY OF THE INFLUENCE ELECTROSTATIC GENERATOR

By Alfred W. Simon

Abstract

A rigorous mathematical theory of the static machine is given and applied in detail to a simple four element machine of a type exemplified by the Kelvin replenisher. It is shown that the potentials of each of the inductors after n cycles satisfies an equation of the form $V_n = C_1 r_1^n + C_2 r_2^n$, where r_1 and r_2 are constants which depend only on the coefficients of capacity and the coefficients of induction of the elements in two configurations, and C_1 and C_2 are constants which depend both on the initial values of the potentials of the inductors and the coefficients of capacity and of induction of the elements in the two determining configurations. The values of the various constants for a non-symmetrical machine, and also for a symmetrical machine are given in tables. It is shown that theoretically the potential produced by an ordinary static machine will increase without limit, approach a constant value, or decrease to zero, depending on the phase at which the brushes are grounded.

A RIGOROUS quantitative theory of the influence electrostatic generator has never been given. The nearest approach is that made by Maxwell,[1] who has given a mathematical treatment of the Kelvin replenisher. However, the theory which he has presented is only an approximation; for he has assumed that the carriers give up all their charges to the inductors, and, further, he has neglected the fact (1) that when a carrier is grounded near one inductor it is also influenced by the other, and the fact (2) that each inductor lowers the potential of the other. Maxwell obtains the result that the potential will tend to increase without limit. V. E. Johnson,[2] who has done a great deal of excellent experimental work on the static machine, recently questions the commonly made inference that the potential produced by a static machine will rise until the increase is balanced by the leakage, and seems to be of the opinion that even if all the leakage could be stopped, the potential would not increase without limit. It was to determine whether this is so as well as to supply a more rigorous theory of the static machine that the following research was undertaken.

Method of Attack

The problem has been attacked in a series of steps. (1) The equations for the charges on the elements of the machine have been written in terms of the potentials, the coefficients of capacity, and the coefficients

[1] Maxwell, Electricity and Magnetism, 3d. edition (1892,) **1**, 110

[2] V. E. Johnson, High Speed Influence Machines, London (1922)

of induction of the elements, for each of those configurations of a complete cycle in which a change of charge occurs on any of the elements, and also for one configuration of the next cycle. To these equations were added a number of other equations involving the charges only and usually expressing conservation of quantity of electricity. (2) from this first basic set of equations, all the charges were eliminated, and a number of simultaneous equations were obtained, each of which gives the potential of a particular element in a particular configuration of the $(n+1)$st cycle in terms of the potentials of a certain number of the elements in the same configuration of the nth cycle. (3) from this second set, a number of additional equations were deduced giving the potentials of the same elements in the same configuration of the $(n+2)$nd, $(n+3)$rd . . . $(n+m)$th cycle. The number m is equal to the number of independent variables in the basic set of equations. Stated in another way, m is equal to the difference between the number of unknowns and the number of relations in the basic set of equations. (4) From the latter set, by elimination, a final set of equations was deduced, in each of which there appears only the potential of any one element assumed in a particular configuration over a number of successive cycles, and each of which is of the form

$$V_{n+m}+k_1V_{n+m-1}+k_2V_{n+m-2}+\cdots+k_mV_n=0\,.$$

This expresses the potential of the element in the form of a recurrent sequence which can be evaluated by the theory of finite differences,[3] and this solves the problem. It must be added that the evaluation requires the solution of an algebraic equation of the mth order, namely, the equation:

$$r^m+k_1r^{m-1}+k_2r^{m-2}+\cdots+k_m=0\,.$$

In practice it is found that the roots of this equation can usually be determined by inspection from the constants k_1, k_2, etc., so that the solution presents no particular difficulty.

In order to make the process clear it will now be applied to the solution of a four element static machine, of a type exemplified by the multiplier of Belli,[4] the machine of Varley,[5] or the replenisher of Lord Kelvin.[6]

Theory of the Replenisher

Consider a machine, represented diagrammatically in Fig. 1, consisting of four elements, two inductors 2, 3 and two carriers 1, 4 which operates

[3] G. Boole, A Treatise on the Calculus of Finite Differences, Chapter XI.

[4] Graetz, Handbuch der Elektrizität und Magnetismus **1**, 38 (1917)

[5] C. F. Varley, Patent Specification, London 1860.

[6] Sir William Thomson, Reprint of Papers on Electrostatics and Magnetism, p. 321, 1872.

as follows: In the configuration a, the carriers are grounded, each receiving a charge opposite to that of the inductor near which it happens to be; these charges are carried forward and shared with the inductors in configuration b; and then the whole process is repeated, 1 and 4 simply exchanging places during the next half cycle. This assumes partial symmetry as regards two successive half cycles, but not necessarily symmetry during any one configuration.

In setting up the equations we shall use the following notation: V_{an} represents the potential of the inductor 2 in configuration a after n cycles; P_{an} that of the inductor 3 under the same conditions; Q_{2an} represents the charge on the inductor 2 in configuration a after n cycles; Q_{3an} the charge of 3 after n cycles, and so on.

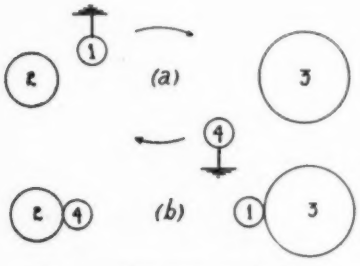

Fig. 1. Non-symmetrical four-element machine

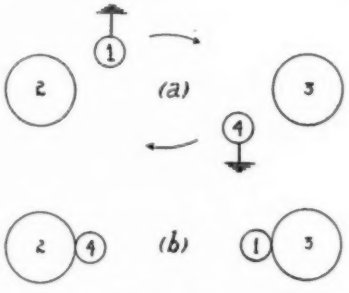

Fig. 2. Symmetrical four-element machine

Our basic set of equations for this machine is then

$$\left.\begin{array}{llll} Q_{1an}=a_{21}V_{an}+a_{31}P_{an}\ ; & Q_{1bn}=b_{13,1}P_{bn}+b_{24,1}V_{bn}\ ; & & \\ Q_{2an}=a_{22}V_{an}+a_{32}P_{an}\ ; & Q_{2bn}=b_{13,2}P_{bn}+b_{24,2}V_{bn}\ ; & & \\ Q_{3an}=a_{23}V_{an}+a_{33}P_{an}\ ; & Q_{3bn}=b_{13,3}P_{bn}+b_{24,3}V_{bn}\ ; & & \\ Q_{4an}=a_{24}V_{an}+a_{34}P_{an}\ ; & Q_{4bn}=b_{13,4}P_{bn}+b_{24,4}V_{bn}\ ; & & \\ Q_{2an+1}=a_{22}V_{an+1}+a_{32}P_{an+1}\ ; & Q_{1an}+Q_{3an}=Q_{1bn}+Q_{3bn}\ ; & Q_{2an+1}=Q_{2bn}\ ; & \\ Q_{3an+1}=a_{23}V_{an+1}+a_{33}P_{an+1}\ ; & Q_{2an}+Q_{4an}=Q_{2bn}+Q_{4bn}\ ; & Q_{3an+1}=Q_{3bn}\ . & \end{array}\right\} \quad (1)$$

In this set there are 14 equations and 16 unknowns; hence by elimination we can derive two simultaneous equations of the form:

$$V_{an+1}=LV_{an}-MP_{an} \quad (2)$$

$$P_{an+1}=SP_{an}-RV_{an} \quad (3)$$

where L, M, S, R, are certain combinations of the a, b coefficients, appearing in the Eqs. (1).

If we solve (2) for P_{an} we have

$$P_{an}=LV_{an}/M-V_{an+1}/M \quad (4)$$

and, raising the subscripts by unity, we have:

$$P_{an+1}=LV_{an+1}/M-V_{an+2}/M$$

Substituting these values in (3) we obtain an equation of the form

$$V_{an+2}-KV_{an+1}+NV_{an}=0 \tag{5a}$$

where $K=(L+S)$; $N=(LS-RM)$.

Similarly, if we eliminate the V's instead of the P's, we obtain

$$P_{an+2}-KP_{an+1}+NP_{an}=0\,. \tag{5b}$$

The theory of finite differences applied to (5a) gives for V_{an} an equation of the form

$$V_{an}=C_1r_1{}^n+C_2r_2{}^n \tag{6a}$$

where r_1 and r_2 are the roots of the quadratic equation

$$r^2-Kr+N=0$$

and C_1 and C_2 are constants which are to be determined from the initial values of V_{an}, i.e. from V_{a1} and V_{a2}. Similarly, from (5b) we obtain

$$P_{an}=C_3r_1{}^n+C_4r_2{}^n \tag{6b}$$

TABLE I

$$\begin{aligned}
L\cdot\Delta a\cdot\Delta b&=+(a_{33}b_{24,2}-a_{32}b_{24,3})(a_{2,24}b_{13,13}-a_{2,13}b_{13,24})\\
&\quad+(a_{33}b_{13,2}-a_{32}b_{13,3})(a_{2,13}b_{24,24}-a_{2,24}b_{24,13})\\
S\cdot\Delta a\cdot\Delta b&=+(a_{22}b_{24,3}-a_{23}b_{24,2})(a_{3,24}b_{13,13}-a_{3,13}b_{13,24})\\
&\quad+(a_{22}b_{13,3}-a_{23}b_{31,2})(a_{3,13}b_{24,24}-a_{3,24}b_{24,13})\\
M\cdot\Delta a\cdot\Delta b&=-(a_{33}b_{24,2}-a_{32}b_{24,3})(a_{3,24}b_{13,13}-a_{3,13}b_{13,24})\\
&\quad+(a_{33}b_{13,2}-a_{32}b_{13,3})(a_{3,13}b_{24,24}-a_{3,24}b_{24,13})\\
R\cdot\Delta a\cdot\Delta b&=-(a_{22}b_{24,3}-a_{32}b_{24,2})(a_{2,24}b_{13,13}-a_{2,13}b_{13,24})\\
&\quad-(a_{22}b_{13,3}-a_{23}b_{31,2})(a_{2,13}b_{24,24}-a_{2,24}b_{24,13})
\end{aligned}$$

$$\begin{aligned}
\Delta a&\equiv(a_{22}a_{33}-a_{23}a_{32})\\
\Delta b&\equiv(b_{13,13}b_{24,24}-b_{13,24}b_{24,13})
\end{aligned}$$

$$\begin{aligned}
2r_1&=-(L+S)+\sqrt{LS-RM}\\
2r_2&=-(L+S)-\sqrt{LS-RM}
\end{aligned}$$

$$\begin{aligned}
C_1&=[V_0(r_2-L)+MP_0]/(r_2-r_1)\\
C_2&=[V_0(r_1-L)+MP_0]/(r_2-r_1)\\
C_3&=[P_0(r_2-S)+RV_0]/(r_2-r_1)\\
C_4&=[P_0(r_1-S)+RV_0]/(r_2-r_1)
\end{aligned}$$

The values of the constants $L,S,M,R,r_1,r_2,C_1,C_2,C_3$, and C_4, for a non-symmetrical machine (Fig. 1) are given in Table I, and for a symmetrical machine (Fig. 2) in Table II.

In these Tables it will be noticed that expressions of the form $b_{pq,rs}$ occur. These expressions are defined by

$$b_{pq,rs}\equiv b_{pr}+b_{ps}+b_{qr}+b_{qs}\,.$$

In particular,

$$b_{13,13}=b_{11}+b_{13}+b_{31}+b_{33};$$
$$b_{24,13}=b_{21}+b_{23}+b_{41}+b_{43}.$$

Written in this form their physical significance is at once apparent;[7] for example, $b_{13,13}$ is the coefficient of capacity of the pair of bodies 1,3; $b_{24,13}$ is the coefficient of induction of the pair 2,4 and the pair 1,3; etc. This notation proves to be a very convenient one for calculation, the symbol b being treated as an operator.

TABLE II

$$L_s\cdot\Delta a_s\cdot\Delta b_s=S_s\cdot\Delta a\cdot\Delta b_s=+(a_{22}a_{2,24}-a_{23}a_{2,13})(b_{24,2}b_{24,24}-b_{24,3}b_{24,13})$$
$$+(\ a_{23}a_{24}-a_{22}a_{21}\ )(b_{24,2}b_{24,13}-b_{24,3}b_{24,24})$$
$$M_s\cdot\Delta a_s\cdot\Delta b_s=R_s\cdot\Delta a_s\cdot\Delta b_s=-(a_{22}a_{2,24}-a_{23}a_{2,13})(b_{24,3}b_{24,24}-b_{24,2}b_{24,13})$$
$$-(\ a_{23}a_{24}-a_{22}a_{21}\ )(b_{24,3}b_{24,13}-b_{24,2}b_{24,24})$$

$$r_{1s}=\frac{(a_{2,24}+a_{2,13})\ (b_{24,2}+b_{24,3})}{(a_{22}+a_{23})\ (b_{24,24}+b_{24,13})}$$

$$r_{2s}=\frac{(a_{2,24}-a_{2,13})\ (b_{24,2}-b_{24,3})}{(a_{22}-a_{23})\ (b_{24,24}-b_{24,13})}$$

$$\Delta a_s\equiv(a_{22}^2-a_{23}^2)$$
$$\Delta b_s\equiv(b_{24,24}^2-b_{24,13}^2)$$

$$+C_{1s}=(V_0+P_0)/2r_{1s})=+C_{3s}$$
$$+C_{2s}=(V_0-P_0)/2r_{2s})=-C_{4s}$$

In order to answer the question raised at the beginning of this article as well as to show the further application of these formulas, let us consider the special case of a symmetrical machine in which the inductor 2 is originally at a voltage V_0 and the inductor 3 at zero potential. Let the arrangement of the elements be that represented diagrammatically in Fig. 2. It is required to determine whether the potentials of the inductors 2 and 3 will increase without limit. Referring to the formula for V_{an} namely,

$$V_{an}=C_{1s}r_{1s}^n+C_{2s}r_{2s}^n$$

we see that if either of the roots r_{1s}, r_{2s} is greater than unity, V_{an} approaches infinity as n increases without limit. Now r_{1s} and r_{2s} can be written in the form

$$r_{1s}=\frac{1+\dfrac{a_{24}+a_{21}}{a_{22}+a_{23}}}{1+\dfrac{b_{24,4}+b_{24\,1}}{b_{24,2}+b_{24\,3}}}\equiv\frac{1+A_1}{1+B_1}:$$

[7] Maxwell, loc. cit.,[1] p. 321

$$r_{2s}=\frac{1+\dfrac{a_{24}-a_{21}}{a_{22}-a_{23}}}{\dfrac{b_{24,4}-b_{24,1}}{b_{24,2}-b_{24,3}}}\equiv\frac{1+A_2}{1+B_2}.$$

Selecting r_{1s} for further analysis we know

$$\begin{aligned}&(a_{24}+a_{21})<0\ ;\\&(a_{22}+a_{23})>0;\\&(b_{24,4}+b_{24,1})>0;\\&(b_{24,2}+b_{24,3})>0.\end{aligned}\tag{7}$$

The first two relations follow directly from the facts that every coefficient of induction is negative and that every coefficient of capacity is positive and greater than any coefficient of induction. In this paper those cases where the inductors completely surround the carriers are excluded.

The last two relations can be proved as follows

$$(b_{24,4}+b_{24,1})=b_{44}+b_{24}+b_{14}+b_{21}\ .$$

For symmetry $b_{21}=b_{34}$, so that we may write

$$(b_{24,4}+b_{24,1})=b_{44}+b_{14}+b_{24}+b_{34}\ ;$$

but we know from electrostatic theory

$$b_{44}>|b_{14}+b_{24}+b_{34}|\ ;$$

hence

$$(b_{24,4}+b_{24,1})>0.$$

Similarly, it can be shown that $(b_{24,2}+b_{24,3})>0$.

Applying the relations (7) to the formula for r_{1s}, we find r_{1s} is positive under all conditions; and the same can be shown to be true of r_{2s}. Further analysis shows that r_{1s} is always less than unity, so that the term $C_{1s}r_{1s}^{n}$ becomes negligible for high values of n. In regard to r_{2s} we can distinguish the following three cases

Case (1). $A_2>B_2$; $r_{2s}>1$. This is the ordinary case as represented diagrammatically in Fig. 1. Under these conditions *the potential of the inductors increases without limit as n increases.*

Case (2). $A_2=B_2$; $r_{2s}=1$. This is an interesting case. For these conditions we have

$$V_{an}=C_{1s}r_{1s}^{n}+C_{2s}\ .$$

We see that as n approaches infinity, V_{an} approaches a constant value, namely, C_{2s}.

Case (3). $A_2<B_2$; $r_{2s}<1$. This case is represented in Fig. 3. Under these conditions, the potentials of the inductors are eventually reduced to zero, the action of the machine being simply to discharge the inductors.

Another result, which can readily be inferred from Eqs. (6a) and (6b) together with Table II, is that in a symmetrical machine the potentials

of the inductors eventually become equal in value but opposite in sign, for the terms involving r_1^n ultimately vanish and the potentials of the inductors 2 and 3 for high values of n are given respectively by

$$V_{an} = C_{2s} r_{2s}^n \; ; \quad P_{an} = -C_{2s} r_{2s}^n \; .$$

The terms involving r_1^n might by analogy be called transients; and it will be noted that if the potentials of the inductors are equal in value but

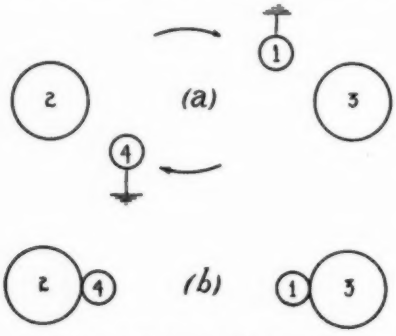

Fig. 3. Self discharging machine.

opposite in sign to begin with, the transient terms are absent altogether, since for this case

$$C_{1s} = C_{3s} = 0 \; .$$

In conclusion I wish to thank Mr. J. A. Shohat of the Mathematics Department for pointing out to me the application of the theory of finite differences, and Professor A. H. Compton for the interest he has shown in this work.

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August 1, 1924.

BOOK REVIEWS

Fourth International Congress of Refrigeration, London, June, 1924. By H. KAMERLINGH ONNES.—This volume covers the matter presented by Prof. Onnes in his capacity as President of the International Institute of Refrigeration. There are some 25 titles covering scientific investigational work carried out at Leiden or in some way in collaboration with the Leiden Laboratory. The articles are generally rather full summaries, in English or in French, of work which has been or will be published in full elsewhere. Ten of the papers are concerned with the study of the properties of gases at low temperatures, so continuing a line of investigation pursued at Leiden for many years. Liquid-vapor conditions, the thermometer scale and fixed points, superconductivity magnetic properties,—all at low temperatures—are all represented.

Probably the most interesting paper is that by H. Kammerlingh Onnes in which is described their latest work on superconductivity. The outstanding discovery is that the current in a superconducting material (lead) is confined to its initial path, even though subjected to considerable outside force tending to displace it in the conductor. It suggests at once that the electrons are in some way restrained from leaving their paths. The fact should prove of considerable importance in both electron and electric theory. Eduard Jjdo, Leiden, 1924.

J. R. ROEBUCK

The Theory of Optics. By SCHUSTER and NICHOLSON. This is the third edition of a book by Schuster appearing in its first edition in 1904 and in a second edition in 1909. Originally a thorough introduction to the field of optical theory, the present work is much improved by numerous modifications and the addition of some new material. Greater preciseness and clarity is, on the whole, the important result secured by the changes made in the discussion of previous topics. Several new section headings are incorporated in the original chapters. Among the topics thus added are Lummer Plates, Earth Tides, Heterochromatic Photometry, Measurements of Star Diameters, Series Spectra, Aether drift, and the Michelson and Morley Experiment.

Two additional chapters presumably written largely by Nicholson have been included, in the first of which the application of the quantum theory to emission spectra has been briefly but on the whole clearly discussed. The reader who has had no previous acquaintance with the principles discussed under the heading Black Body Radiation will doubtless find the arguments in that section hard to follow. As noted by the authors it is little more than an outline of the subject. The introduction here of the phrase "conditionally periodic system" (p. 361) the meaning of which is not explained until late in the next chapter is unfortunate. The last chapter deals with the Dynamical Theory of Spectra. It is a concise treatment of an extensive subject and will doubtless serve better as an excellent summary for the initiated rather than as an introduction for the beginner.—Pp. XV+397. Price $5.50, Longmans, Green and Co., 1924.

R. C. GIBBS

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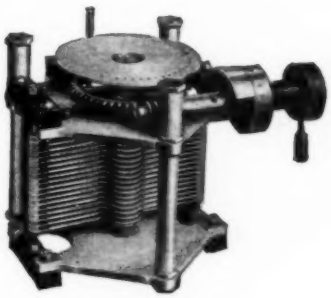

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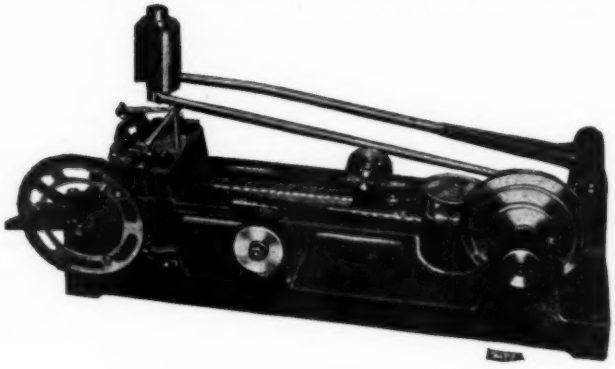

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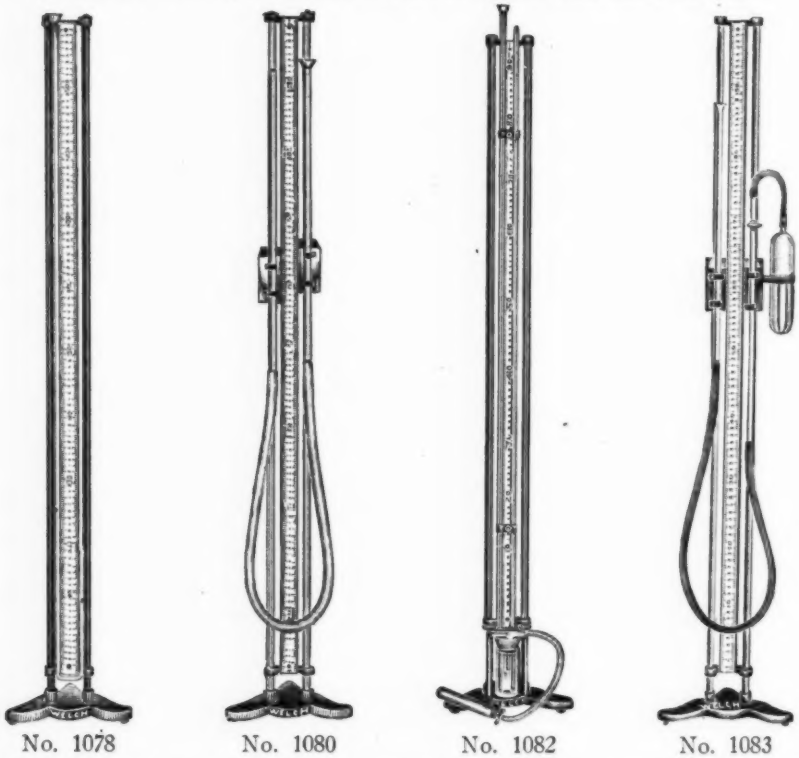

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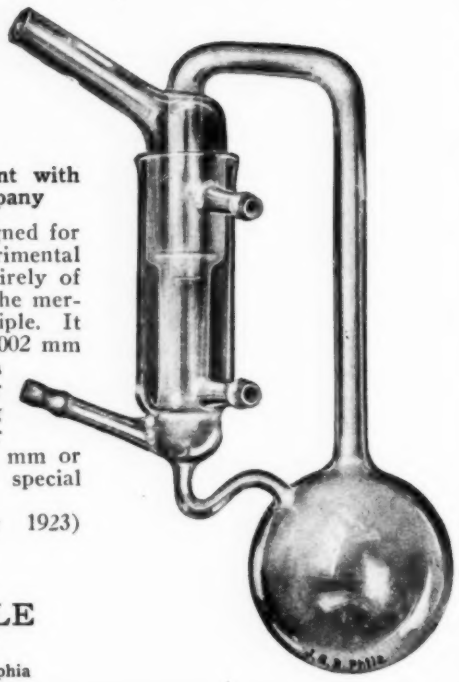

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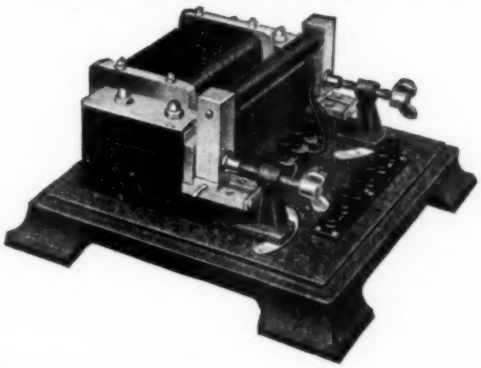

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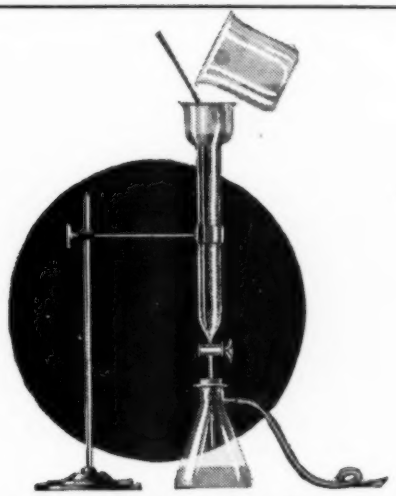

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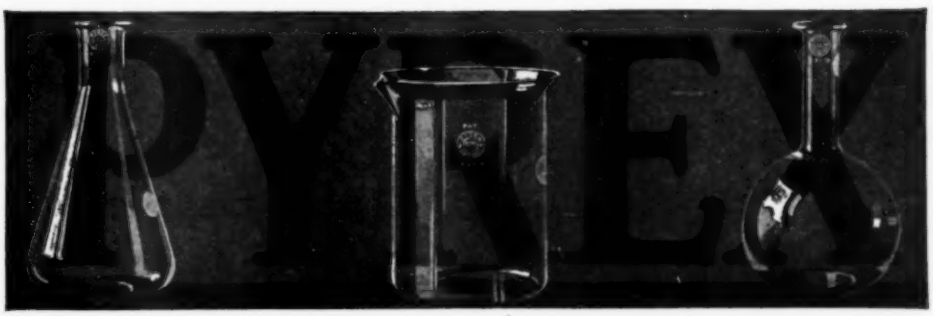

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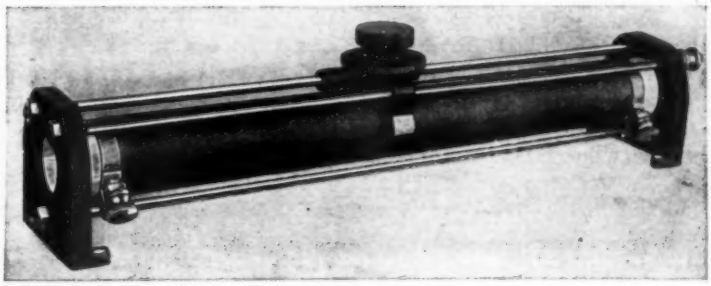

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